IS 8161-7 (1977): Guide for equipment reliability testing, Part 7: Compliance test plans for failure rate and mean time between failures assuming constant failure rate [LITD 2: Reliability of Electronic and Electrical Components and Equipment]

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IS 8161 (Part 7) : 2012
IEC 61124 : 2006

भारतीय मानक

उपस्कर विश्वसनीयता परीक्षण के लिए मार्गदर्शिका

भाग 7 स्थिर विफलता दर एवं स्थिर विफलता तीव्रता के लिए अनुपालन परीक्षण

(पहला पुनरीक्षण)

Indian Standard

GUIDE FOR EQUIPMENT RELIABILITY TESTING

PART 7 COMPLIANCE TESTS FOR CONSTANT FAILURE RATE AND CONSTANT FAILURE INTENSITY

(First Revision)

ICS 19.020; 03.120.30; 21.020

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Reliability of Electronic and Electrical Components and Equipment Sectional Committee, LITD 02

NATIONAL FOREWORD

This Indian Standard (First Revision) which is identical with IEC 61124 : 2006 'Reliability testing — Compliance tests for constant failure rate and constant failure intensity' issued by the International Electrotechnical Commission (IEC) was adopted by the Bureau of Indian Standards on the recommendation of the Reliability of Electronic and Electrical Components and Equipment Sectional Committee and approval of the Electronics and Information Technology Division Council.

The text of IEC Standard has been approved as suitable for publication as an Indian Standard without deviations. Certain conventions are, however, not identical to those used in Indian Standards. Attention is particularly drawn to the following:

a) Wherever the words 'International Standard' appear referring to this standard, they should be read as 'Indian Standard'.
b) Comma (,) has been used as a decimal marker while in Indian Standards, the current practice is to use a point (.) as the decimal marker.

In this adopted standard, reference appears to certain International Standards for which Indian Standards also exist. The corresponding Indian Standards which are to be substituted in their respective places are listed below along with their degree of equivalence for the editions indicated:

International Standard	*Corresponding Indian Standard*	*Degree of Equivalence*
IEC 60050 (191) : 1990 International Electrotechnical Vocabulary (IEV) — Chapter 191: Dependability and quality of service	IS 1885 (Part 39) : 1999 Electrotechnical vocabulary: Part 39 Dependability of electronic and electrical items (*second revision*) IS 1885 (Part 45) : 1977 Electrotechnical vocabulary: Part 45 Capacitors	Technically Equivalent
IEC 60605-2 Equipment reliability testing — Part 2: Design of test cycles	IS 8161 (Part 2) : 1986 Guide for equipment reliability testing: Part 2 Design of test cycles	do
IEC 60605-3 (All parts) Equipment reliability testing — Part 3: Preferred test conditions	IS 8161 (Part 3/Sec 1 and 2) : 1986 Guide for equipment reliability testing: Part 3 Preferred test conditions for equipment reliability testing, Section 1 Indoor portable equipment (low degree of simulation) Section 2 Equipment for stationary use in weather protected locations (high degree of simulation)	do
IEC 60605-4 : 2001 Equipment reliability testing — Part 4: Statistical procedures for exponential distribution — Point estimates, confidence intervals, prediction intervals and tolerance intervals	IS 8161 (Part 4) : 1985 Guide for equipment reliability testing: Part 4 Procedure for determining point estimates and confidence limits from equipment reliability determination tests	do

International Standard	*Corresponding Indian Standard*	*Degree of Equivalence*
IEC 60605-6 Equipment reliability testing — Part 6: Tests for the validity of the constant failure rate or constant failure intensity assumptions	IS 8161 (Part 6/Sec1 and 2) : 1983 Guide for equipment reliability testing : Part 6 Tests for validity of a constant failure rate assumption, Section 1 Chi-square test Section 2 Kolmogorov-Smirnov test	Technically Equivalent

The technical committee has reviewed the provisions of the following International Standards referred in this standard and has decided that they are acceptable for use in conjunction with this standard:

International Standard	*Title*
IEC 60300-3-5 : 2001	Dependability management — Part 3-5: Application guide — Reliability test conditions and statistical test principles
IEC 61123 : 1991	Reliability testing — Compliance test plans for success ratio

Only the English language text in the International Standard has been retained while adopting it in this Indian Standard and as such the page numbers given here are not the same as in the IEC Publication.

For the purpose of deciding whether a particular requirement of this standard is complied with, the final value, observed or calculated, expressing the result of a test or analysis, shall be rounded off in accordance with IS 2 : 1960 'Rules for rounding off numerical values (*revised*)'. The number of significant places retained in the rounded off value should be the same as that of the specified value in this standard.

Indian Standard

GUIDE FOR EQUIPMENT RELIABILITY TESTING

PART 7 COMPLIANCE TESTS FOR CONSTANT FAILURE RATE AND CONSTANT FAILURE INTENSITY

(First Revision)

1 Scope

This International Standard gives a number of optimized test plans, the corresponding operating characteristic curves and expected test times. In addition the algorithms for designing test plans using a spreadsheet program are also given, together with guidance on how to choose test plans.

This standard specifies procedures to test whether an observed value of

– failure rate,
– failure intensity,
– mean time to failure (MTTF),
– mean operating time between failures (MTBF),

conforms to a given requirement.

It is assumed, except where otherwise stated, that during the accumulated test time, the times to failure or the operating times between failures are independent and identically exponentially distributed. This assumption implies that the failure rate or failure intensity is constant.

Four types of test plans are described as follows:

– truncated sequential tests;
– time/failure terminated tests;
– fixed calendar time terminated tests without replacement;
– combined test plans.

This standard does not cover guidance on how to plan, perform, analyse and report a test. This information can be found in IEC 60300-3-5.

This standard does not describe test conditions. This information can be found in IEC 60605-2 and in IEC 60605, Parts 3-1 to 3-6.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050(191), *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

IEC 60300-3-5:2001, *Dependability management – Part 3-5: Application guide – Reliability test conditions and statistical test principles*

IEC 60605-2, *Equipment reliability testing – Part 2: Design of test cycles*

IEC 60605-3 (all parts), *Equipment reliability testing – Part 3: Preferred test conditions*

IEC 60605-4:2001, *Equipment reliability testing – Part 4: Statistical procedures for exponential distribution – Point estimates, confidence intervals, prediction intervals and tolerance intervals*

IEC 60605-6, *Equipment reliability testing – Part-6: Tests for the validity of the constant failure rate or constant failure intensity assumptions*

IEC 61123:1991, *Reliability testing – Compliance test plans for success ratio*

3 Terms, definitions, symbols and acronyms

3.1 Terms and definitions

For the purposes of this document the terms and definitions given in IEC 60050(191) apply.

The terms “failure rate” and “failure intensity” are used as meaning constant failure rate and constant failure intensity.

3.2 Acronyms and symbols

3.2.1 Acronyms

OC Operating characteristic

SPRT Sequential probability ratio test (in some literature called probability ratio sequential test (PRST)

3.2.2 Symbols

The generic symbol λ is used in the following for failure rate and failure intensity.

The symbol m is used to denote both the following reliability measures:

– mean operating time between failures, MTBF;

– mean time to failure, MTTF.

When used, the relationship between the above quantities, under the given assumptions, is:

$$\lambda = \frac{1}{m}$$

Sequential test plans (see Clause 6) and fixed time/failure terminated test plans (see Clause 7) are based on m as a reliability measure, thus in these cases:

$$m = \frac{1}{\lambda}$$

c	acceptable number of failures during the test
D	discrimination ratio; $D = m_0/m_1$ or $D = \lambda_1/\lambda_0$
k	summation variable for failures
MTBF	mean operating time between failures
MTTF	mean time to failure
m	true MTBF or MTTF
m_0	specified MTTF or MTBF $m_0 = 1/\lambda_0$ (design goal)
m_1	lower limit for MTTF or MTBF $m_1 = 1/\lambda_1$
n	number of test items at the beginning of the test
P_a	probability of acceptance
p_0	acceptable failure ratio
q_0	acceptable success ratio, $q_0 = 1 - p_0$
$R(t)$	reliability at time t
r	observed number of failures during the test
r_0	test truncation failure number for sequential tests (SPRT)
T^*	accumulated test time
T_a^*	accumulated test time stated as accept criterion
$T_{a,\,min}^*$	minimum test time for r = 0 stated as accept criterion
T_e^*	expected accumulated test time to decision
$T_e^*(+)$	expected accumulated test time to acceptance
T_r^*	accumulated test time stated as reject criterion
T_t^*	accumulated test time stated as termination criterion
t	test time
t^*	test truncation time
t_t^*	test time for each test item (assumed here to be the same for all test items)
t_i	life time of failed item i
$t_{cal,\,t}^*$	calendar test time stated as termination criterion
$P(r)$	probability of r failures
$\Delta\mu_0$	auxiliary quantity for determination of c, $\Delta\mu_0 = c - \mu_0$
α	nominal producer's risk (type I risk)
α'	true producer's risk (type I risk)

β nominal consumer's risk (type II risk)

β' true consumer's risk (type II risk)

λ true failure rate per item

λ_0 expected failure rate per item (design goal)

λ_1 upper limit for constant failure rate per item

μ expected number of failures during the test at the true λ

NOTE μ is not necessarily an integer.

μ_0 expected number of failures during the test at the specified λ_0 as parameter of the Poisson distribution $\mu_0 = \lambda_0 T_t^*$

4 General requirements and area of application

4.1 Requirements

It is assumed, except where otherwise stated, that during the accumulated test time the times to failure or the operating times between failures are independent, and identically exponentially distributed. This assumption implies that the failure rate or failure intensity is constant. Under this assumption there is no difference between failure rate and failure intensity. Therefore they are both called λ and referred to in the following as failure rate.

It is assumed that the requirement is specified in terms of the acceptable constant failure rate λ_0, or the acceptable mean number of failures per time unit, λ_0 or the acceptable mean time to failure or mean operating time between failures, m_0.

If it is necessary to test the constant failure rate/constant failure intensity assumption, the procedures given in IEC 60605-6 should be used.

4.2 Applicability to replaced and repaired items

The sequential test plans (see Clause 6), the time/failure terminated test plans (see Clause 7) and the combined test plans (see Clause 10) are applicable to the following:

– replacement of failed items,
– without replacement of failed items,

under the assumption that

– an item can be replaced by repair of the item itself;
– the accumulated test time is calculated as elapsed operating item-time, in accordance with 5.4;
– replaced items belong to the same population as the original items;
– repaired items can be considered to have the same failure intensity after repair as they had before they failed.

The calendar time/failure terminated test plans in Clause 9, however, are applicable to cases where failed items are not replaced and where a fixed number of items are placed on test for a fixed calendar time. This means that the test is running, even though the number of items under test may not remain constant because some items may not survive.

4.3 Types of test plans

4.3.1 General

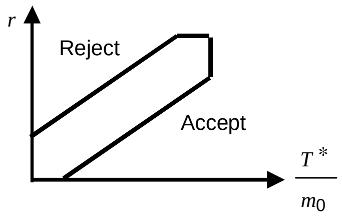
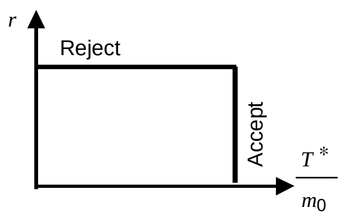
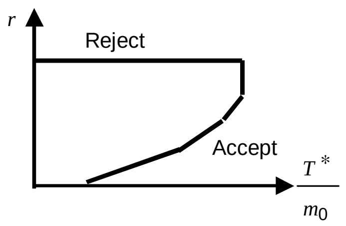
Test plans are given for four types of tests:

- truncated sequential tests (see Clause 6);
- time/failure terminated tests (for fixed time/failure terminated tests see Clause 7, and for alternative time/failure terminated tests, see Clause 8);
- calendar time terminated tests without replacement (see Clause 9);
- combined test plans (see Clause 10).

4.3.2 Advantages and disadvantages of the different test plan types

The different test plan types are shown in Table 1.

Table 1 – Advantages and disadvantages for the different test plan types

Test plan type	Advantages	Disadvantages
Truncated sequential Test plans in Clause 6 (Table 2), Annex A and Annex D 	Assures highest test efficiency (shortest test time). For the A test plans the user can develop new test plans using a spreadsheet program The C test plans have true alpha and beta risks which are closer to the nominal risks than the A test plans	Complicated to administer For marginal reliability, the test time is longer than for time/failure terminated and combined test plans The test plans in Annex D are made by an iterative process. They cannot be computed using the spreadsheets described in Annex F
Time/Failure terminated Test plans in Clause 7 (Table 3) and Annex B 	Simple to understand and to administer For marginal reliability, the test time is often shorter than for sequential test plans. The user can develop new test plans using a spreadsheet program. More failure information is collected for a Test, Analyse and Fix Program (TAAF) using these test plans	For very reliable, or very unreliable test items, the test time is much longer than for the sequential test plans, or for the combined test plans (in case of high reliability)
Combined Test plans in Clause 10 (Table 4) and Annex D 	Combines the advantages of the time/failure terminated test plans and the sequential test plans. Very reliable test items will be accepted fast while test items with early failures will not be rejected early in the test. More failure information is collected for a Test, Analyse and Fix Program (TAAF) using these test plans	More complicated to administer For test items of marginal reliability, the test may last longer than a sequential test plan The test plans in Annex D are made by an iterative process. They cannot be computed using the spreadsheets described in Annexes F and H

5 General test procedure

5.1 Test conditions

A guide on how to plan, perform, analyse and report a test can be found in IEC 60300-3-5. Test conditions can be found in IEC 60605-2 and IEC 60605-3-1 to IEC 60605-3-6.

All of these test types can be accelerated to accommodate high reliability requirements and enable shorter time to market. Mission and life-cycle environment profiles and test conditions are provided as follows:

The mission profiles should be used to determine the environmental specifications and should be derived from the operational life profile defined by the equipment or system requirements. If this information is not provided in the original contractual documentation, provision should be made to derive the mission profiles and the equipment environmental specifications.

This derivation should make use of historical data on similar equipment applications and mounting platforms and the effect of equipment location in the platform should be accounted for. Each significant life-cycle event has to be considered including transportation, handling, installation and testing as well as for platform category and operational conditions.

Environmental test conditions (reliability growth, qualification, acceptance tests) should be performed under combined influence of electrical power input, temperature, vibration, humidity and other appropriate test conditions:

– the test level for these test conditions should be derived from the equipment's mission and environment profiles;
– when the equipment is designed for one application with a single mission or one type of repetitive mission, there is a simple relationship between the test profile, mission profile and life-cycle environmental profile. The tests conditions should simulate the stress level during the mission;
– if the equipment is designed for multi-missions and environmental conditions, the test profile should represent a composite of those missions, with the test levels and durations being rated according to the percentage of each mission type expected during the equipment's life-cycle. In order to derive realistic test conditions and levels, the actual environments (especially temperature and vibration) should be measured at the location where the equipment is to be mounted during actual operation.

If design changes are made during the test, the accumulated test time shall start from 0 after the changes have been incorporated in the test items, unless specifically agreed between the parties and justified in the test report (see IEC 60300-3-5).

5.2 General characteristics of the test plans

All test plans and decision criteria are based on either of two characteristics, as follows:

– an acceptable number of failures in a specified test time; or
– an acceptable test time for a specified number of failures.

The detailed criteria for test plans, with variants, are given in 6.3, 7.3, 8.4 and 9.3.

5.3 Data to be recorded

In order to apply the statistical procedures defined in this standard, the following data shall be recorded:

- the observed number of failures;
- the accumulated test time.

If it is necessary to test the assumption stated in 4.1, the following data shall also be recorded:

- the time of occurrence of each failure.

5.4 Calculation of accumulated test time, T^*

Whenever the term "time" is used in this standard it can be replaced by other similar measures of the duration of the operation for the item(s) under test, e.g. distance driven, cycles, starts and test cycles (see IEC 60605-2).

The accumulated test time T^* is calculated as the sum of the test time accumulated by each item in the test. For more than one repaired item under test, the accumulated test time is the sum of all elapsed operational times (excluding all repair and other down times).

When *n* items are put on test at the same time, the accumulated test time can be determined using the following formulae:

$T^* = n \times t^*$ for tests with replacement of failed items

$T^* = (n-r)t^* + \sum_{i=1}^{r} t_i$ for tests without replacement of failed items

where

n is the number of items at the beginning of the test;

t^* test truncation time;

t_i life time of failed items;

r number of failed items.

It is necessary to monitor the test items during the test in order to replace failed items, register accumulated test time, and where required record time to failures (see 5.3)

Several samples can be tested simultaneously or sequentially. The minimum sample size is $n \geq 1$ with replacement and $n \geq r_0 + 1$ without replacement.

Calendar time is used to state the test time when more than one item is on test simultaneously (see Clause 9).

For guidance on how to calculate accumulated test time when items enter the test after the test has started, or are removed from the test before the test ends (censoring) see IEC 60300-3-5 and IEC 60605-4 (Annex C).

The accumulated test time is normally counted as the operating time. If the parties to the test wish to include other operational modes such as, for example, “idle” or “stand by” in the test time e.g. in a test cycle, this has to be specified and agreed before the test is started (see IEC 60605-2).

5.5 Number of failures

In this standard the test result is determined by the test time and the number of failures. But in practice failures can have very different consequences. Therefore, the user should consider if there are failures that are so critical that the test is failed even though the total number of failures are not exceeded. Alternatively, there can be failures where the consequences are so insignificant that they should not be considered failures. All failures are counted except when they can be classified as non relevant, e.g. caused by factors not relevant for the reliability of the item in the field. Such classification rules should be agreed between the parties before start of the test (see IEC 60300-3-5). Re-classification of failures requires agreement between the parties and shall be justified in the test report.

6 Sequential test plans

6.1 General

Sequential test plans are characterized by decision rules for accepting or rejecting compliance or continuing the test at any test time. They are determined by selected values of risks and discrimination ratio. To implement a test plan, m_0 shall be specified. An overview of test plans is given in 6.4. The test plans and the corresponding operating characteristic curves and expected test times are stated in Annex A and Annex D. Examples and a mathematical reference are given in Annex E. Sequential test plans derived using a different methodology based on GOST 27.402 [1]1 are numbered C1 to C8 and are included in Annex D. In order to facilitate comparison between the C test plans and the combined test plans they have been plotted together in Annex D. Descriptions of the iterative procedure for creating those test plans can be found in Annex K.

6.2 Common test procedure

The test procedure should include the following steps:

- decide the combined environmental conditions to be applied during the test based on analysis to determine appropriate test conditions (see 5.1 and IEC 60300-3-5);
- specify m_0, according to requirement or agreement;
- choose risks, α and β, and discrimination ratio, D, considering that low values of α, β, and D make a more powerful test, but require longer test time and/or more items;
- select α and β. These are generally chosen to be equal (shared risk) for easier calculations and agreements;
- select an appropriate test plan from Table 2 and Annex A or D;
- note the test times, T_a^* for accept, and T_r^* for reject;
- perform the test and apply the decision criteria in accordance with 6.3.

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1 Figures in square brackets refer to the Bibliography.

6.3 Decision criteria

The accumulated test time and the number of failures shall be compared with the accept and reject criteria continuously during the test, or at intervals not greater than the length of the test cycle or the monitoring interval (see 5.1 and 7.1.3 of IEC 60300-3-5). The following criteria shall apply:

Accept the test — if the elapsed $T^* \geq T_a^*$ for the observed $r \leq r_0$.
The specified requirement is regarded as being complied with.

Continue testing — if the elapsed T^* is between the limits $T_r^* < T^* < T_a^*$ for the observed $r \leq r_0$.
No decision can be taken.

Reject the test — if the elapsed $T^* \leq T_r^*$ for the observed $r \leq r_0$, or if $r \geq r_0 + 1$.
The specified requirement is regarded as not being complied with.

6.4 Overview of test plans

Table 2 summarizes the sequential test plans given in Annex A and D. These annexes provide, for each test plan, a decision graph, decision table, graphs of operating characteristic curve and expected test time to decision.

Table 2 – Overview of the sequential test plans given in Annex A and D

Test plan No.	Characteristics of the plan			Minimum test time (Acceptance at 0 failures)	Expected time for decision in multiples of m_0 for $m = m_0$	Test truncation time in multiples of m_0	True risks for	
	Nominal risks		Discrimination ratio				$m = m_0$	$m = m_1$
	α %	β %	D	$\frac{T^*_{a,min}}{m_0}$	$\frac{T^*_e}{m_0}$	$\frac{T^*_t}{m_0}$	α' %	β' %
A.1	10	10	1,5	4,39	17,88	33,04	12,03	9,77
A.2	10	10	2	2,20	5,37	10,30	13,39	9,62
A.3	10	10	3	1,098	1,77	3,152	15,09	9,43
A.4	10	10	5	0,549	0,65	1,102	16,81	9,24
A.5	20	20	1,5	2,77	8,00	14,37	24,32	18,92
A.6	20	20	2	1,39	2,31	4,73	27,27	18,18
A.7	20	20	3	0,693	0,72	1,54	31,03	17,24
A.8	30	30	1,5	1,69	2,93	5,41	37,70	26,70
A.9	30	30	2	0,847	0,79	1,91	43,24	24,32
C.1	5	5	1,7	7,584 0	23,89	29,300 0	5,00	5,00
C.2	10	5	1,7	5,032 9	16,57	24,240 0	9,998	5,00
C.3	10	10	1,7	4,429 4	13,28	18,030 0	10,00	9,999
C.4	15	5	1,7	5,836 6	16,40	19,900 0	14,999	5,001
C.5	20	10	1,7	4,023 2	10,00	12,600 0	19,998	10,002
C.6	20	20	1,7	2,342 4	5,62	8,300 0	19,992	20,016
C.7	30	20	1,7	2,102 1	3,95	5,500 0	29,990	20,014
C.8	30	30	1,7	1,275 7	2,13	3,800 0	30,056	29,912

7 Fixed time/failure terminated test plans – Fixed duration test plans

7.1 General

Fixed time/failure terminated test plans are characterized by decision rules for accepting or rejecting compliance when the test time for termination has been reached, or the acceptable number of failures has been exceeded. The test plans are determined by selected values of risks and discrimination ratio. To implement a test plan, m_0 shall be specified (or derived).

There are four types of fixed duration tests:

a) time terminated tests with replacement;

b) time terminated tests without replacement;

c) failure terminated tests with replacement;

d) failure terminated tests without replacements.

Test plans are given in 7.4. Operating characteristics and expected test times are given in Annex B, examples and a mathematical reference are given in Annex G.

7.2 Common test procedure

The test procedure should include the following steps:

– decide the combined environmental conditions to be applied during the test based on analysis to determine appropriate test conditions (see 5.1 and IEC 60300-3-5);

– specify m_0 according to requirement or agreement;

– choose risks, α = β (shared risk) or α not equal to β, and discrimination ratio, *D*, considering that low values of α, β and *D* make a more powerful test, but require longer time and/or more items. It is important that α, β and *D* is selected together considering the risk for the producer, the risk for the consumer as well as m_1 determined by *D*;

– select an appropriate test plan from Table 3, consulting Annex B;

– determine the test time for termination, T_t^*, and the acceptable number of failures, *c*;

– perform the test and apply the decision criteria according to 7.3.

7.3 Decision criteria

The following decision criteria shall apply:

Accept the test	if r ≤ c at T_t^*. The specified requirement is regarded as being complied with.
Reject the test	if r > c at or before T_t^*. The specified requirement is regarded as not being complied with.

7.4 Test plans

Table 3 summarizes the recommended time/failure terminated test plans. The test time is accumulated until either a predetermined amount of test time has been exceeded (accept), or a predetermined number of failures has occurred (reject). Graphs of operating characteristic curves and expected accumulated test time to decision are given in Annex B.

Table 3 – Fixed time/failure terminated test plans

Test plan No.	Characteristics of the plan			Test time for termination	Acceptable number of failures	True risks for	
	Nominal risks		Discrimination ratio			$m = m_0$	$m = m_1$
	α %	β %	D	T_t^*/m_0	c	α' %	β' %
B.1	5	5	1,5	54,10	66	4,96	4,84
B.2	5	5	2	15,71	22	4,97	4,99
B.3	5	5	3	4,76	8	5,35	5,40
B.4	5	5	5	1,88	4	4,25	4,29
B.5	10	10	1,5	32,14	39	10,00	10,20
B.6	10	10	2	9,47	13	10,00	10,07
B.7	10	10	3	3,10	5	9,40	9,90
B.8	10	10	5	1,08	2	9,96	9,48
B.9	20	20	1,5	14,30	17	19,49	19,94
B.10	20	20	2	3,93	5	20,40	20,44
B.11	20	20	3	1,47	2	18,37	18,40
B.12	30	30	1,5	5,41	6	29,99	29,95
B.13	30	30	2	1,85	2	28,28	28,54

NOTE The test time for termination (in multiples of m_0) is equal to the expected number of failures during the test at the specified λ_0 that is $T_t^*/m_0 = T_t^* \times \lambda_0 = r_0$.

8 Design of alternative time/failure terminated test plans

8.1 General

Alternative time/failure terminated test plans are characterized by decision rules for accepting or rejecting compliance when the test time for termination has been reached or the acceptable number of failures has been exceeded. The test plans are determined by selected values of risks and test time. The test time is chosen independently of λ_0 which enables test plans to be designed individually to meet particular needs including, for example, economy, resources or time scheduling.

The derived discrimination ratio is used to evaluate the test plan. In order to implement a test plan, λ_0 shall be specified (or derived). The transformation of failure rate and mean time to failure or mean operating time between failures to λ_0 is given in 3.2.2.

It is also possible to choose different test plan parameters and determine unknown parameters as indicated in Annex C and Annex I.

Graphs for determining test plan parameters are given in Annex C, including the associated operating characteristic curves. Examples and a mathematical reference are given in Annex I.

8.2 Design procedures

The procedures given in Annex C may be applied directly to tests in which

– replacement occurs, and replacement/repair times are negligible,

– replacement does not occur, and $\lambda_0 t_t^* < 0{,}1$ is valid.

In such cases, $T_t^* = nt_t^*$.

However, the procedures may also be applied to tests in which

– replacement occurs, and replacement times are long (e.g. by repair),

– replacement does not occur, and $\lambda_0 t_t^* > 0{,}1$.

In such cases, the formulae involving accumulated test time, T_t^* (see 5.4) shall be used, and in the formulae $n t_t^*$ shall be replaced by T_t^*. This implies that n and t^* cannot be separated, as in Clause C.2.

In each case, when applicable, the values of α and β, as stated in Table 2, shall be considered as preferred values.

Annex C describes how test plans are derived and examples are included.

8.3 Common test procedure

The test procedure should include the following steps:

– decide the combined environmental conditions to be applied during the test based on analysis to determine appropriate test conditions (see 5.1 and IEC 60300-3-5)

– specify the acceptable λ_0, according to requirement or agreement;

– choose risks, α and β, test time for each item, t_t^*, and number of test items, n, according to resources;

– design the test plan according to Annex C and I using the figures in Annex C;

– evaluate the test plan especially regarding the derived discrimination ratio D, and perhaps redesign with modified parameters to satisfy the need;

– perform the test with the stated test time, $T_t^* = n t_t^*$ and the derived acceptable number of failures, c, and apply the decision criteria according to 8.4.

8.4 Decision criteria

The following decision criteria shall apply:

Accept the test	if $r \leq c$ at T_t^*, the specified requirement is regarded as being complied with.
Reject the test	if $r > c$ at or before T_t^*, the specified requirement is regarded as not being complied with.

9 Calendar time/failure terminated test plans for non-replaced items

9.1 General

Fixed calendar time/failure terminated test plans for non-replaced items are characterized by decision rules for accepting or rejecting compliance when the test time for termination has been reached, or the acceptable number of failures has been exceeded. The test plans are determined by selected values of risks and discrimination ratios. To implement a test plan, λ_0 shall be specified (or derived). The transformation of m_0 to λ_0 is indicated in 3.2.2.

Since no replacement takes place, the number of items under test will generally not remain constant, and the accumulated time of operation is not relevant. Thus, the compliance test plans for success ratio, Table 2 of IEC 61123, should be used.

However, if the number of items is large compared to the number of failures, the time/failure terminated test plans (see Clause 7) may be used.

Examples and a mathematical reference are given in Annex J.

9.2 Common test procedure

The test procedure should include the following steps:

- decide the combined environmental conditions to be applied during the test based on analysis to determine appropriate test conditions (see 5.1 and IEC 60300-3-5);
- specify λ_0, according to requirement or agreement;
- choose risks, $\alpha = \beta$, and discrimination ratio, D, considering that low values of α, β and D make a more powerful test but require longer time and/or more items;
- choose test termination time, $t^*_{\mathrm{cal,t}}$, or number of items, n, and calculate according to 9.4.2 or 9.4.3, respectively;
- select an appropriate test plan from Table 2 of IEC 61123, using the derived q_0 and the found n, to give the acceptable number of failures c;
- perform the test and apply the decision criteria according to 9.3.

9.3 Decision criteria

The decision criteria include the following:

Accept the test	if $r \leq c$ at $t^*_{\mathrm{cal,t}}$ the specified requirement is regarded as being complied with;
Reject the test	if $r > c$ at or before $t^*_{\mathrm{cal,t}}$ the specified requirement is regarded as not being complied with.

9.4 Use of Table 2 of IEC 61123:1991 for fixed calendar time tests

9.4.1 General

Two procedures may be used. The first one, given in 9.4.2, is used when the (maximum available) test time is given. The second one, given in 9.4.3, is used when the (maximum available) number of items is given.

The following formulae apply:

$$p_0 = 1 - q_0 = 1 - e^{-t/m_0} = 1 - e^{-\lambda_0 t} \tag{1}$$

where $t = t^*_{\mathrm{cal,t}}$

and the derived

$$t^*_{\mathrm{cal,t}} = -\, m_0 \ln(1 - p_0) = -\, m_0 \ln(q_0) \tag{2}$$

where

p_0 is the acceptable failure ratio;

q_0 is the acceptable success ratio, $q_0 = 1 - p_0$.

NOTE 1 According to the normal use of IEC 61123, p_0 or q_0 is specified, but in this case p_0 is derived from the specified m_0 or λ_0, according to Formula (1). For further information on p_0 and q_0, see IEC 61123.

NOTE 2 In this case, p_0 expresses the failure probability 1-$R(t)$ for $t = t^*_{\mathrm{cal,t}}$.

NOTE 3 The procedures can only determine the product $t^* n$.

9.4.2 Procedure when the test time is given

The test procedure should include the following steps:

a) choose $t^*_{\mathrm{cal,t}} \leq 0{,}223\, m_0 = \dfrac{0{,}223}{\lambda_0}$ (to be within the range of Table 2 of IEC 61123);

b) calculate p_0 using Formula (1);

c) adjust $t^*_{\mathrm{cal,t}}$ if necessary to fit a $q_0 = 1 - p_0$ stated in Table 2 of IEC 61123 using Formula (2);

d) select a relevant test plan from Table 2 of IEC 61123 fulfilling the required D and $\alpha = \beta$, and thereby determine n and c.

9.4.3 Procedure when the number of items is given

The test procedure should include the following steps:

a) select a relevant test plan from Table 2 of IEC 61123 fulfilling the required D and $\alpha = \beta$, and with n close to the indicated n, and thereby determine $q_0 = 1 - p_0$, n and c;

b) calculate $t^*_{\mathrm{cal,t}}$ using Formula (2).

10 Combined test plans

10.1 General

The combined test plans use a reject line with a constant number of failures, while the accept line is curved (see Table 1). Therefore, test items with a high reliability will be accepted early while items with many early failures will not be rejected immediately. Test plans are also suitable for a test, an analysis and a fix program (TAAF) as more information is collected about the failure modes than for a sequential test. The test plans are derived using an iterative procedure that is described in Annex K.

10.2 Common test procedure

- decide the combined environmental conditions to be applied during the test based on analysis to determine appropriate test conditions (see 5.1 and IEC 60300-3-5);
- specify m_0, according to requirement or agreement;
- choose risks, α and β , and discrimination ratio, D, considering that low values of α, β and D make a more powerful test, but require longer time and/or more items;
- select an appropriate test plan from Table 4 and Annex *D*;
- note the test times, T_{a}^{*} for accept, and T_{r}^{*} for reject;
- perform the test and apply the decision criteria in accordance with 10.3.

10.3 Decision criteria

The accumulated test time and the number of failures shall be compared with the accept and reject criteria continuously during the test; or at intervals not greater than the length of the test cycle or the monitoring interval (see 5.1 and 7.1.3 of IEC 60300-3-5).

The following criteria shall apply:

Accept the test	if the elapsed $T^{*} \geq T_{\mathrm{a}}^{*}$ for the observed $r \leq r_0$. The specified requirement is regarded as being complied with.
Continue testing	if the elapsed T^{*} is between the limits $T_{\mathrm{r}}^{*} < T^{*} < T_{\mathrm{a}}^{*}$ for the observed $r \leq r_0$. No decision can be taken.
Reject the test	If the number of failures exceeds $r = r_0 + 1$ during the test. The specified requirement is regarded as not being complied with.

10.4 Test plans

Table 4 – Combined test plans in Annex D

Test plan	Characteristics of the plan			Minimum test time (Acceptance at 0 failures)	Expected time for decision in multiples of m_0 for $m = m_0$	Test truncation time in multiples of m_0	Acceptable number of failures
	Nominal risks		Discrimination ratio				
	α %	β %	D	$T^*_{a,min}/m_0$	T^*_e/m_0	T^*_t/m_0	c
D.1	5	5	1,7	6,454 0	22,07	29,463 2	38
D.2	10	5	1,7	4,721 9	16,59	24,517 0	30
D.3	10	10	1,7	4,426 4	13,12	18,183 9	23
D.4	15	5	1,7	5,732 0	16,22	19,989 8	24
D.5	20	10	1,7	3,949 5	9,80	12,757 3	15
D.6	20	20	1,7	2,436 5	5,60	8,568 4	10
D.7	30	20	1,7	2,144 1	3,96	5,714 6	6
D.8	30	30	1,7	1,219 4	2,30	4,388 1	4

11 Performing the test

Guidance on test conditions and practical management of the test can be found in 5.1, IEC 60300-3-5, IEC 60605-2 and the IEC 60605-3-1 to IEC 60605-3-6 series.

12 Presentation of results

When presenting the results, the following information shall be given:

– test conditions (see 5.1 and IEC 60300-3-5);
– identification of test items, number of items on test and possible replacement during test;
– the test plan used;
– specified acceptable λ_0 or m_0 ;
– nominal test plan parameters given or derived, for example risk levels, discrimination ratio, accumulated test time for termination, acceptable number of failures;
– true test plan parameters, if required;
– observed number of failures and justification for any failures classified as non-relevant;
– definition of accumulated test time especially if test cycles or several mission modes are used;
– accumulated test time when the test is stopped;
– result(s) of other analyses, if required;
– conclusions.

Annex A
(normative)

Tables and graphs for sequential test plans

NOTE 1 See clause 6.

NOTE 2 This annex uses the symbols listed in 3.2.

A.1 Test plan A.1 – α = 0,10; β = 0,10; D = 1,5

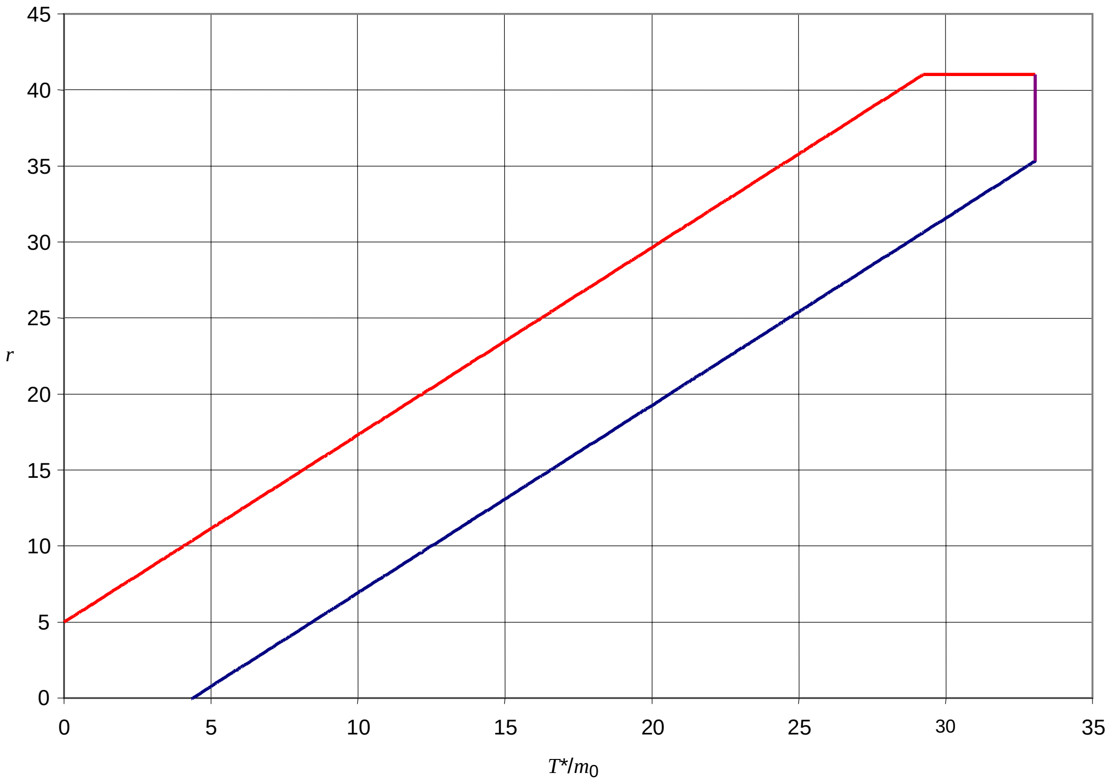

Figure A.1 – Accept and reject lines for test plan A.1

Table A.1 – Accept and reject lines for test plan A.1

r	T_r^* / m_0 **Reject** (equal or less)	T_a^* / m_0 **Accept** (equal or more)	r	T_r^* / m_0 **Reject** (equal or less)	T_a^* / m_0 **Accept** (equal or more)
0	–	4,39	21	13,00	21,42
1	–	5,21	22	13,81	22,23
2	–	6,02	23	14,62	23,05
3	–	6,83	24	15,43	23,86
4	–	7,64	25	16,24	24,67
5	0,025	8,45	26	17,05	25,48
6	0,836	9,26	27	17,87	26,29
7	1,65	10,07	28	18,73	27,10
8	2,46	10,88	29	19,49	27,91
9	3,27	11,69	30	20,30	28,72
10	4,08	12,50	31	21,11	29,53
11	4,89	13,31	32	21,92	30,34
12	5,70	14,13	33	22,73	31,16
13	6,51	14,94	34	23,54	31,97
14	7,32	15,75	35	24,35	32,78
15	8,13	16,56	36	25,16	33,04
16	8,94	17,37	37	25,97	33,04
17	9,76	18,18	38	26,79	33,04
18	10,57	19,00	39	27,60	33,04
19	11,40	19,80	40	28,41	33,04
20	12,19	20,61	r_0 =41	29,22	33,04
			42	33.04	N/A

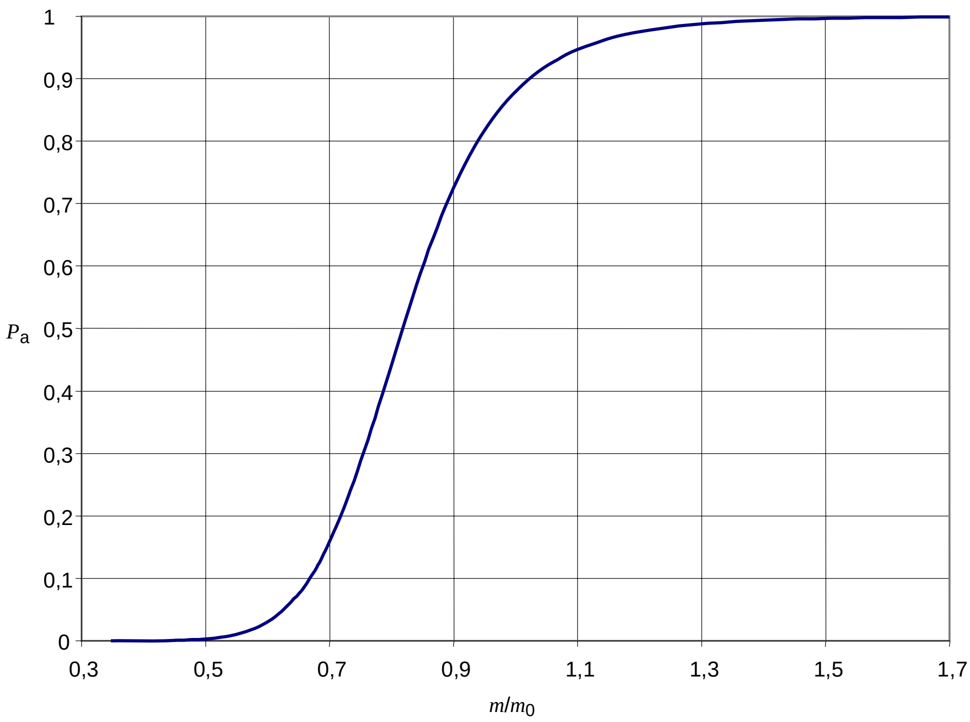

Figure A.2 – Test plan A.1 – Operating characteristic curve

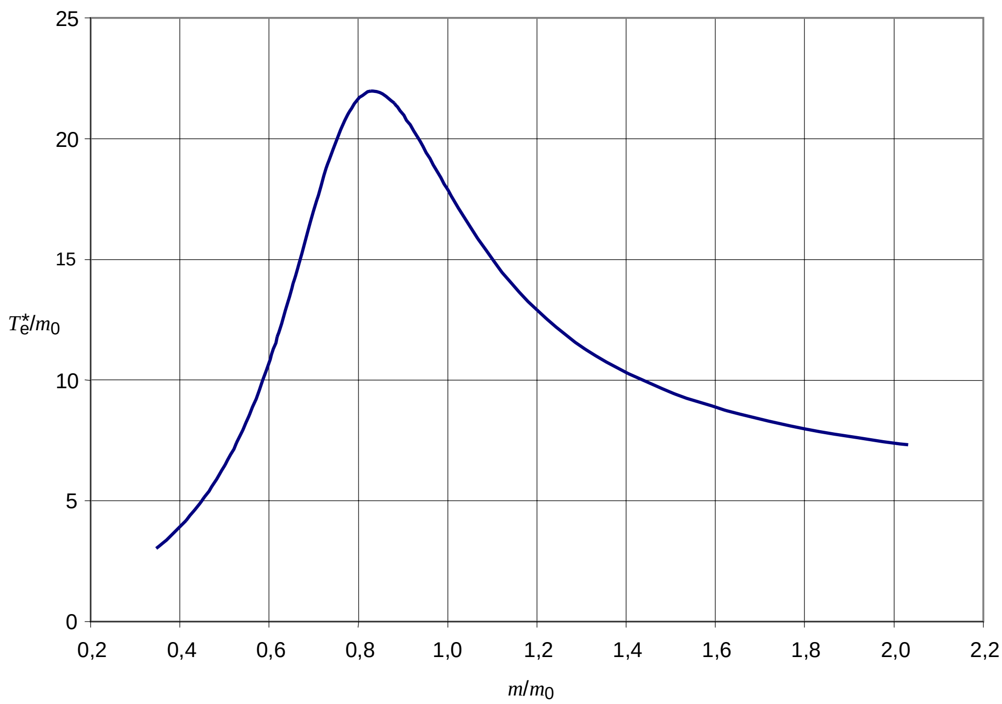

Figure A.3 – Test plan A.1 – Expected accumulated test time to decision

A.2 Test plan A.2 – α = 0,10 β = 0,10 *D* = 2,0

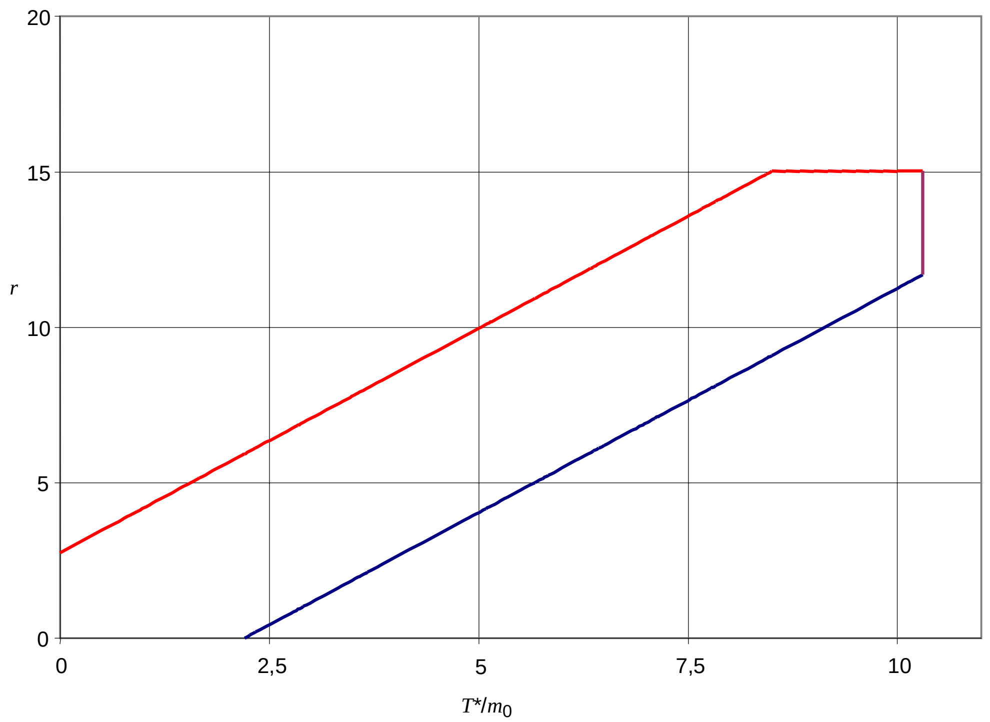

Figure A.4 – Accept and reject lines for test plan A.2

Table A.2 – Accept and reject lines for test plan A.2

r	T_r^* / m_0 **Reject** (equal or less)	T_a^* / m_0 **Accept** (equal or more)
0	–	2,20
1	–	2,89
2	–	3,58
3	0,170	4,28
4	0.863	4,97
5	1,56	5,66
6	2,25	6,36
7	2,94	7,05
8	3.64	7,74
9	4,33	8,44
10	5,02	9,13
11	5,72	9,82
12	6,41	10,30
13	7,10	10,30
14	7,79	10,30
$r_0 = 15$	8,49	10,30
16	10,30	N/A

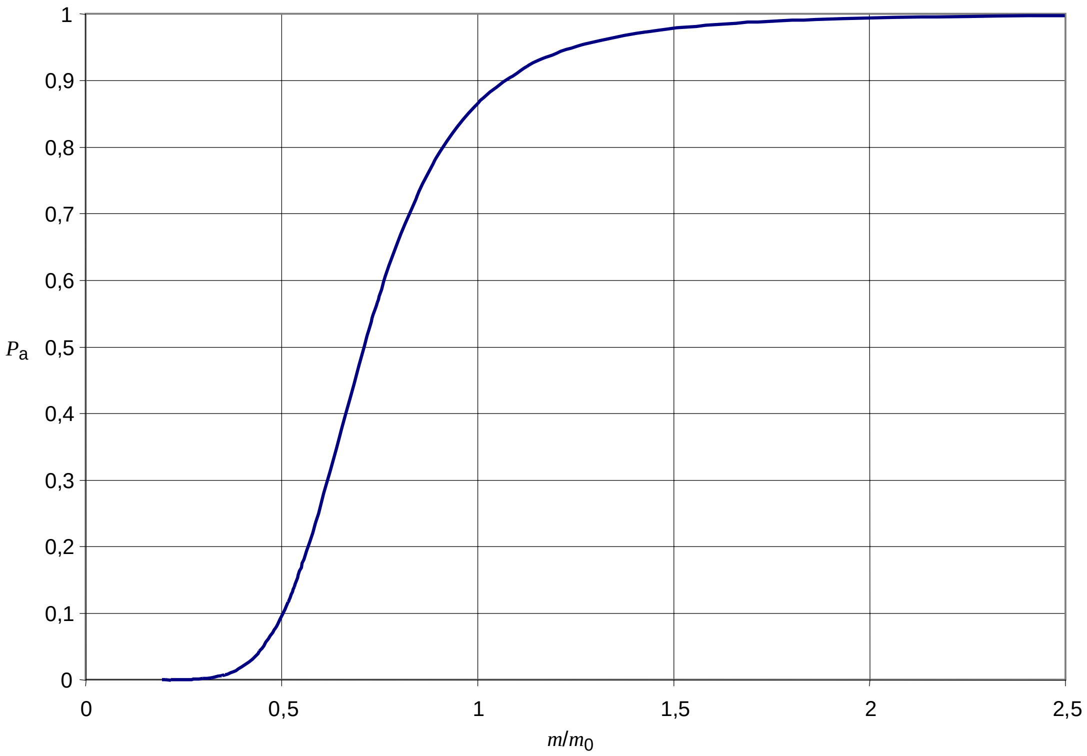

Figure A.5 – Test plan A.2 – Operating characteristic curve

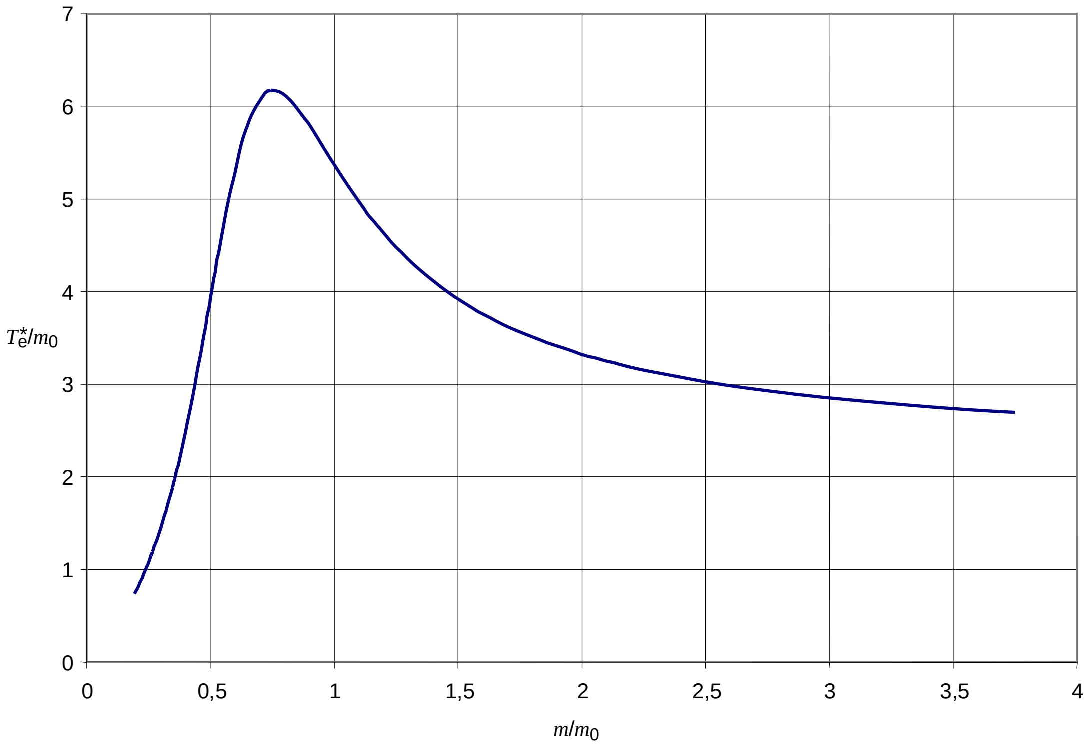

Figure A.6 – Test plan A.2 – Expected accumulated test time to decision

A.3 Test plan A.3 – α= 0,10; β = 0,10; D = 3,0

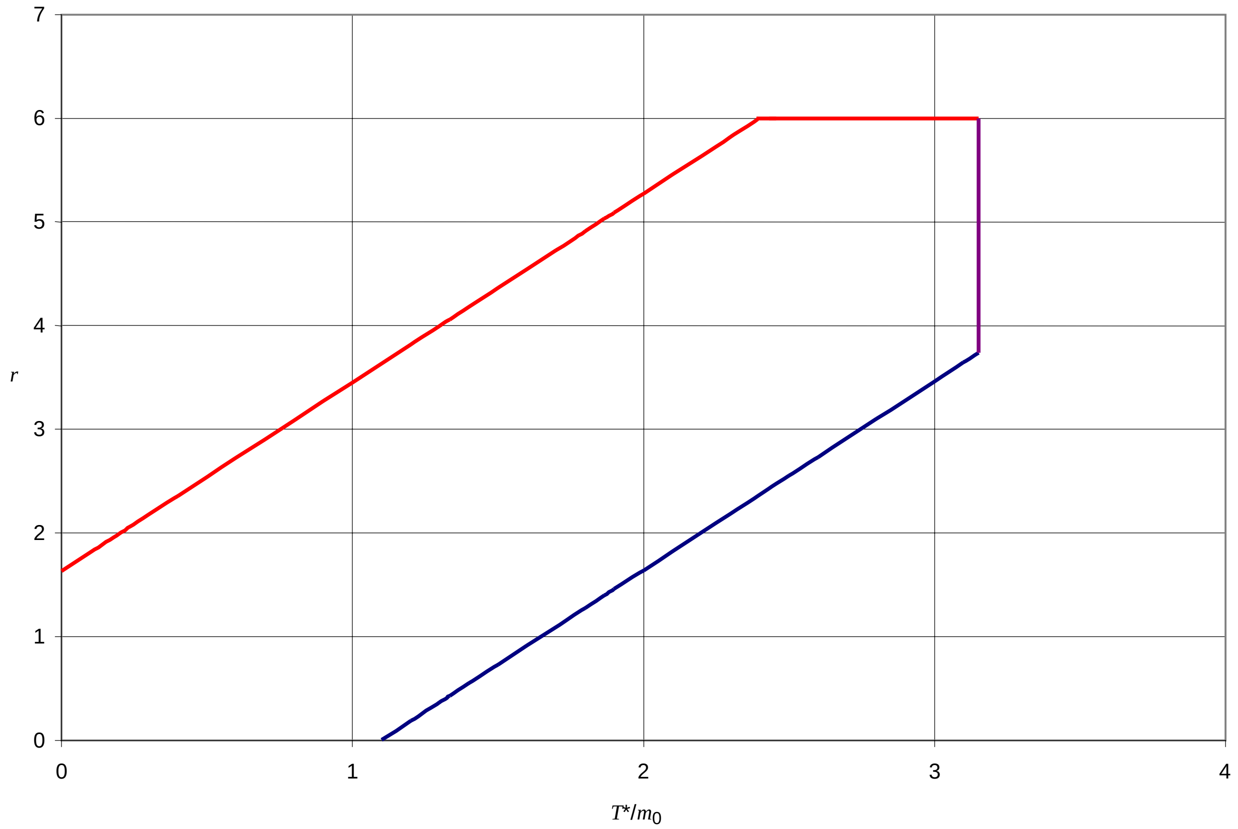

Figure A.7 – Accept and reject lines for test plan A.3

Table A.3 – Accept and reject lines for test plan A.3

r	T_r^* / m_0 **Reject** (equal or less)	T_a^* / m_0 **Accept** (equal or more)
0	–	1,098
1	–	1,648
2	0,203	2,197
3	0,752	2,747
4	1,301	3,152
5	1,851	3,152
$r_0 = 6$	2,400	3,152
7	3,152	N/A

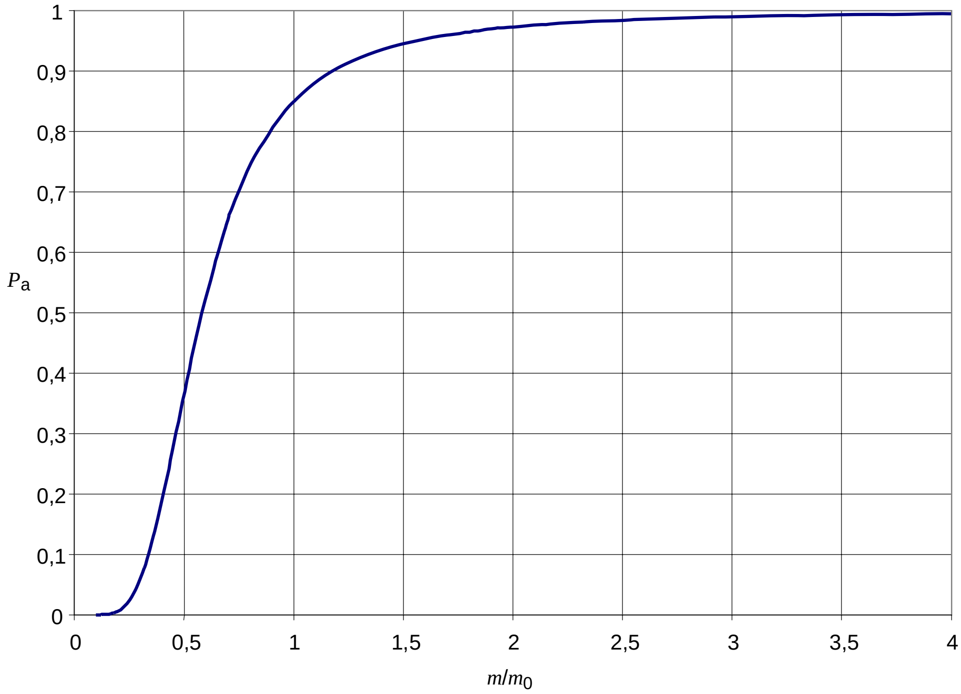

Figure A.8 – Test plan A.3 – Operating characteristic curve

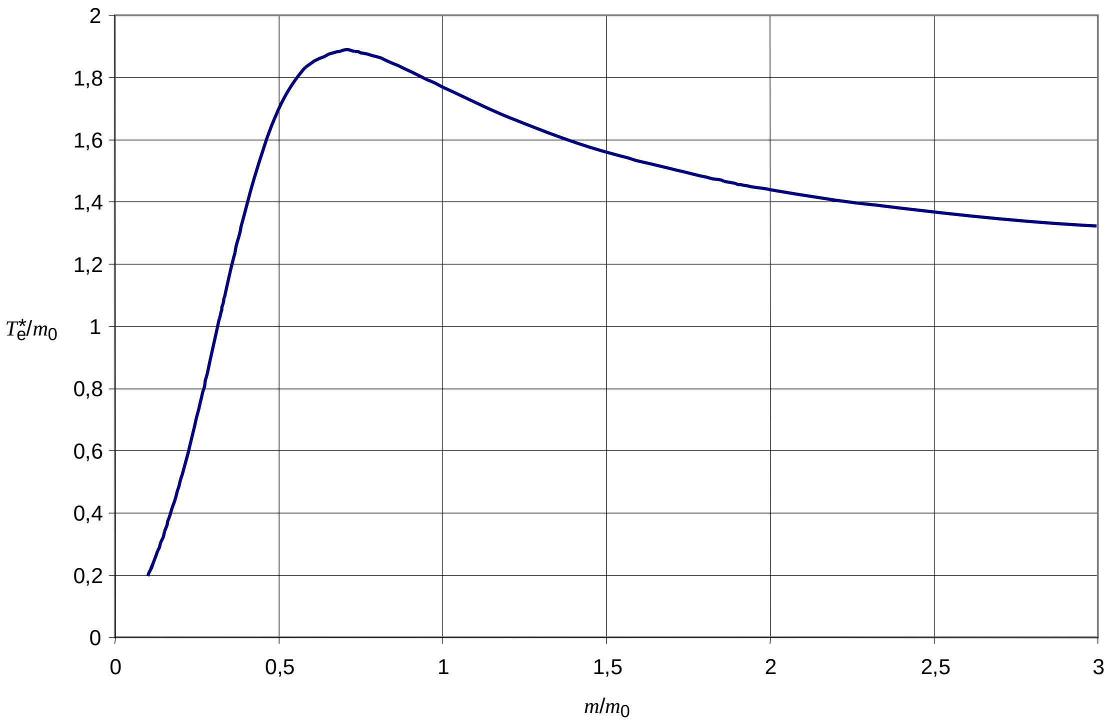

Figure A.9 – Test plan A.3 – Expected accumulated test time to decision

A.4 Test plan A.4 – α = 0,10; β = 0,10; *D* = 5,0

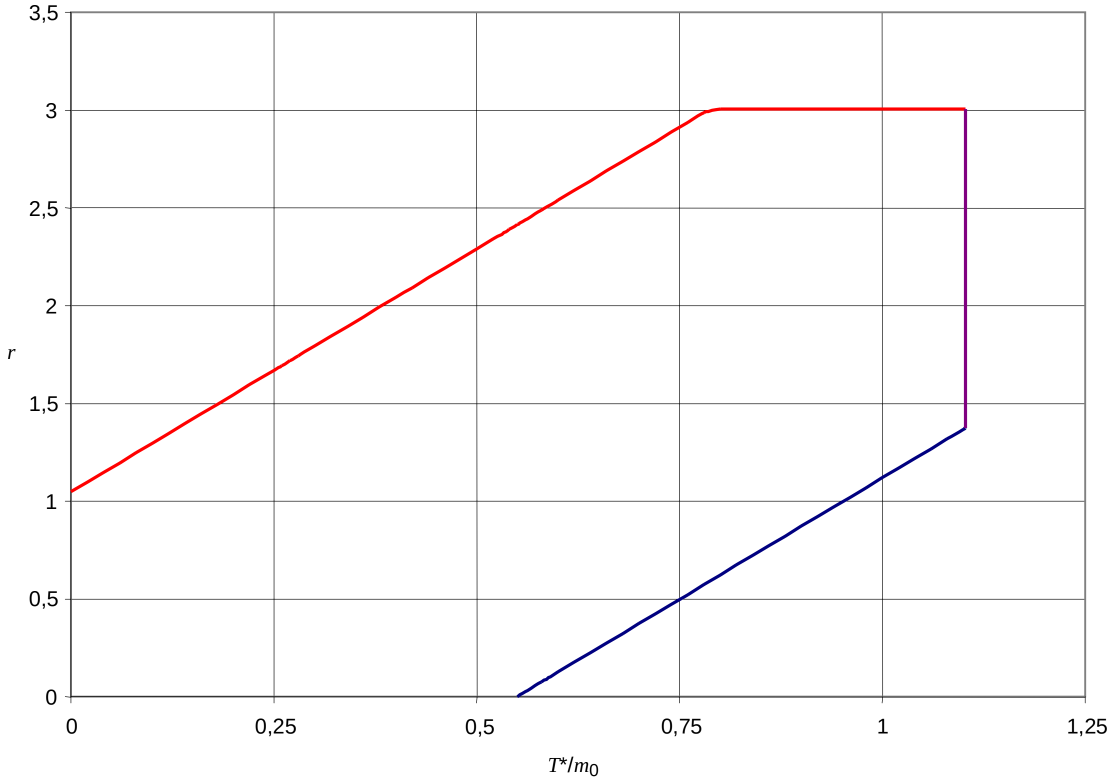

Figure A.10 – Accept and reject lines for test plan A.4

Table A.4 – Accept and reject lines for test plan A.4

r	T_r^* / m_0 **Reject** (equal or less)	T_a^* / m_0 **Accept** (equal or more)
0	–	0,549
1	–	0,951
2	0,383	1,102
$r_0 = 3$	0,785	1,102
4	1,102	N/A

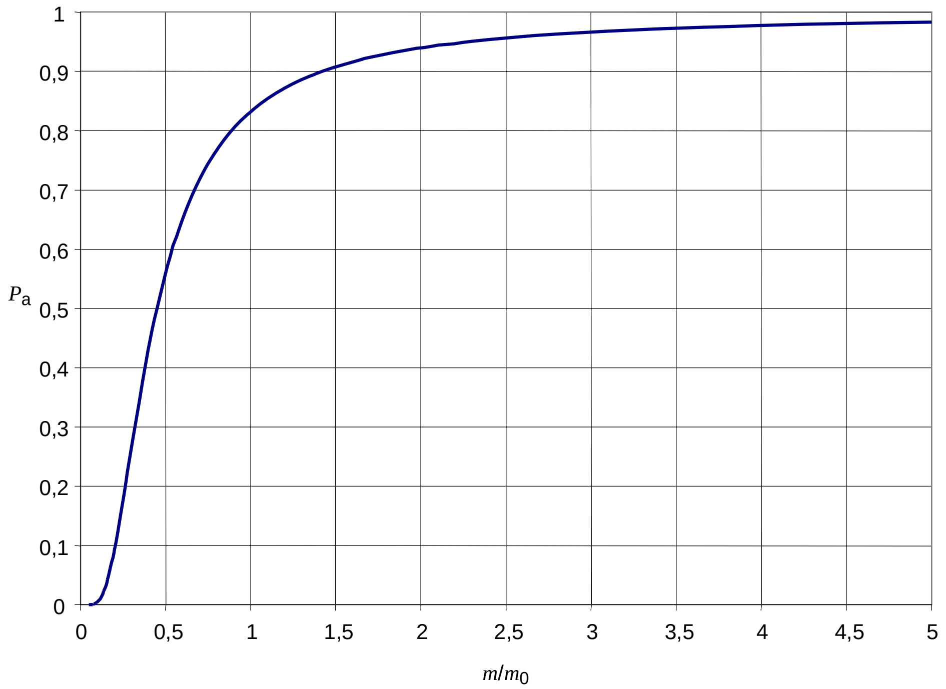

Figure A.11 – Test plan A.4 – Operating characteristic curve

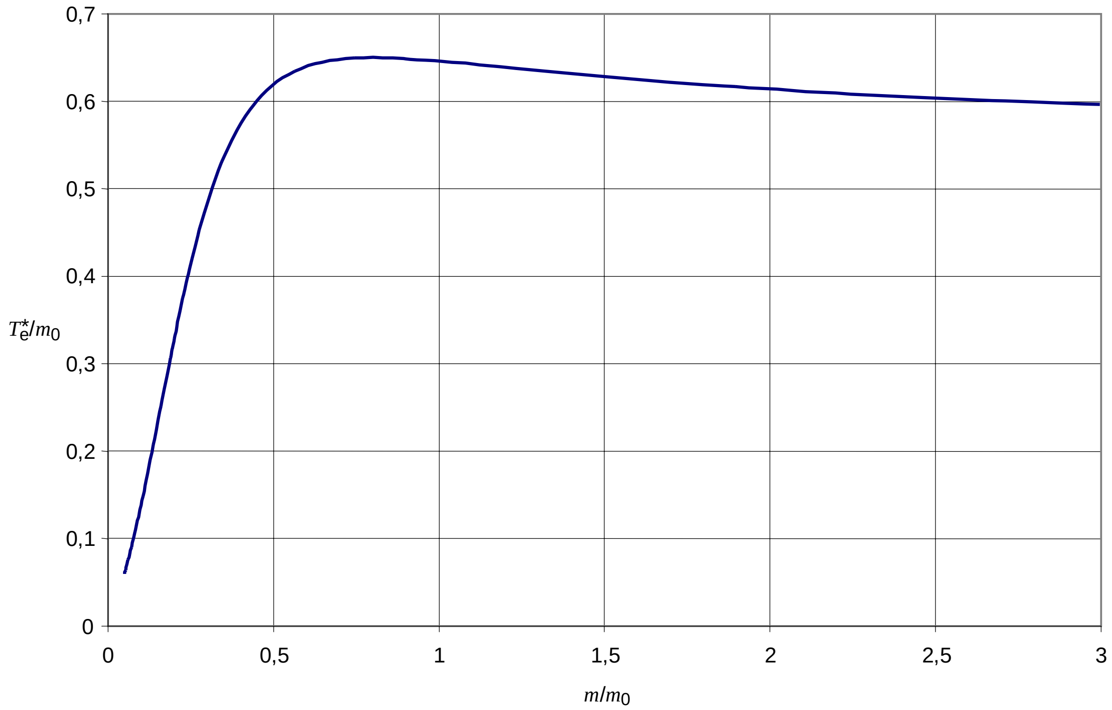

Figure A.12 – Test plan A.4 – Expected accumulated test time to decision

A.5 Test plan A.5 – α = 0,20; β = 0,20; *D* = 1,5

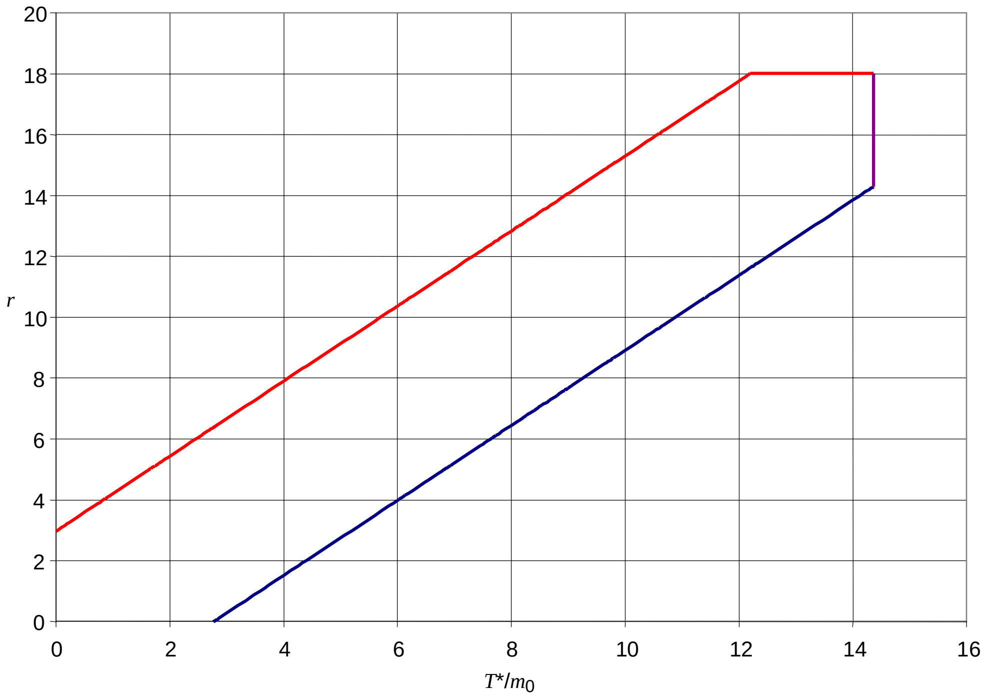

Figure A.13 – Accept and reject lines for test plan A.5

Table A.5 – Accept and reject lines for test plan A.5

r	T_r^* / m_0 **Reject** (equal or less)	T_a^* / m_0 **Accept** (equal or more)
0	–	2,77
1	–	3,58
2	–	4,39
3	0,025	5,21
4	0,836	6,02
5	1,65	6,83
6	2,46	7,64
7	3,27	8,45
8	4,08	9,26
9	4,89	10,07
10	5,70	10,88
11	6,51	11,69
12	7,32	12,50
13	8,13	13,31
14	8,95	14,13
15	9,76	14,37
16	10,57	14,37
17	11,38	14,37
$r_0 = 18$	12,19	14,37
19	14,37	N/A

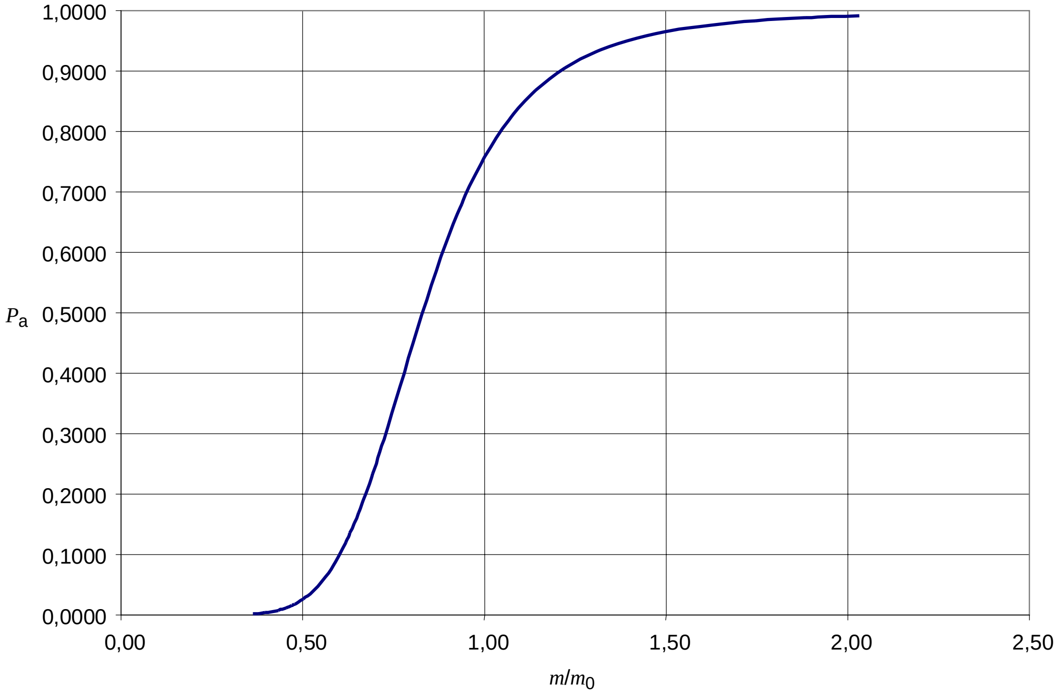

Figure A.14 – Test plan A.5 – Operating characteristic curve

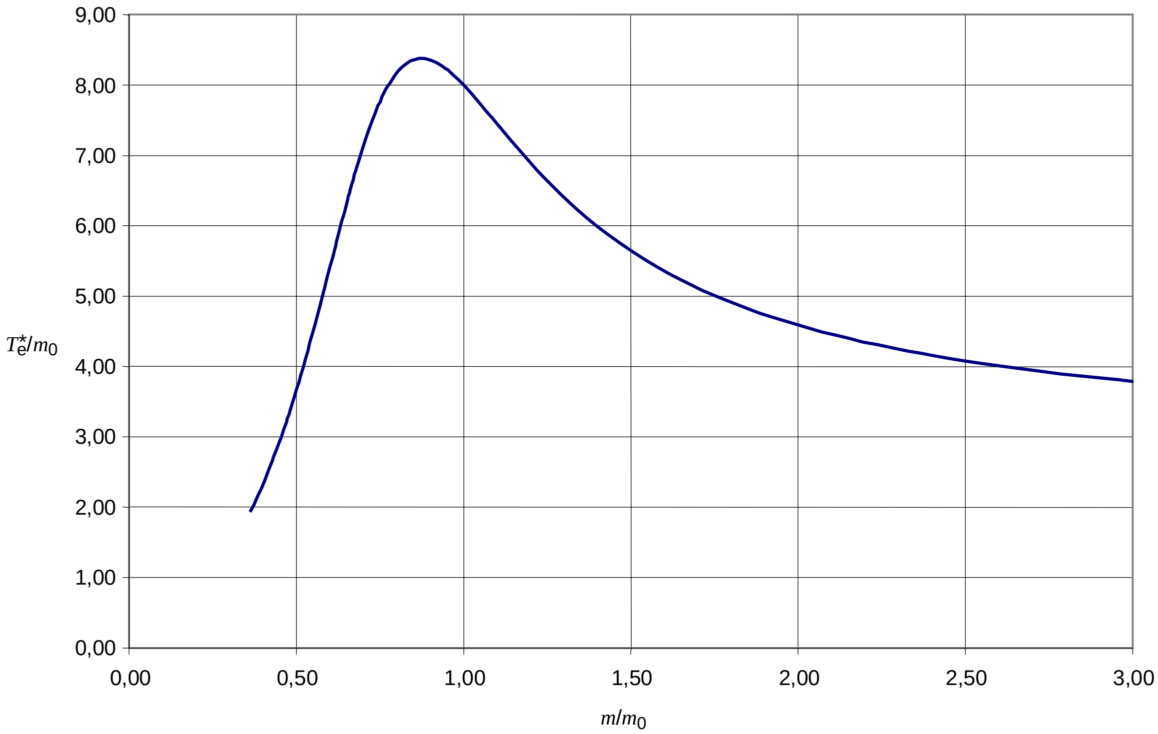

Figure A.15 – Test plan A.5 – Expected accumulated test time to decision

A.6 Test plan A.6 – α = 0,20; β = 0,20; *D* = 2,0

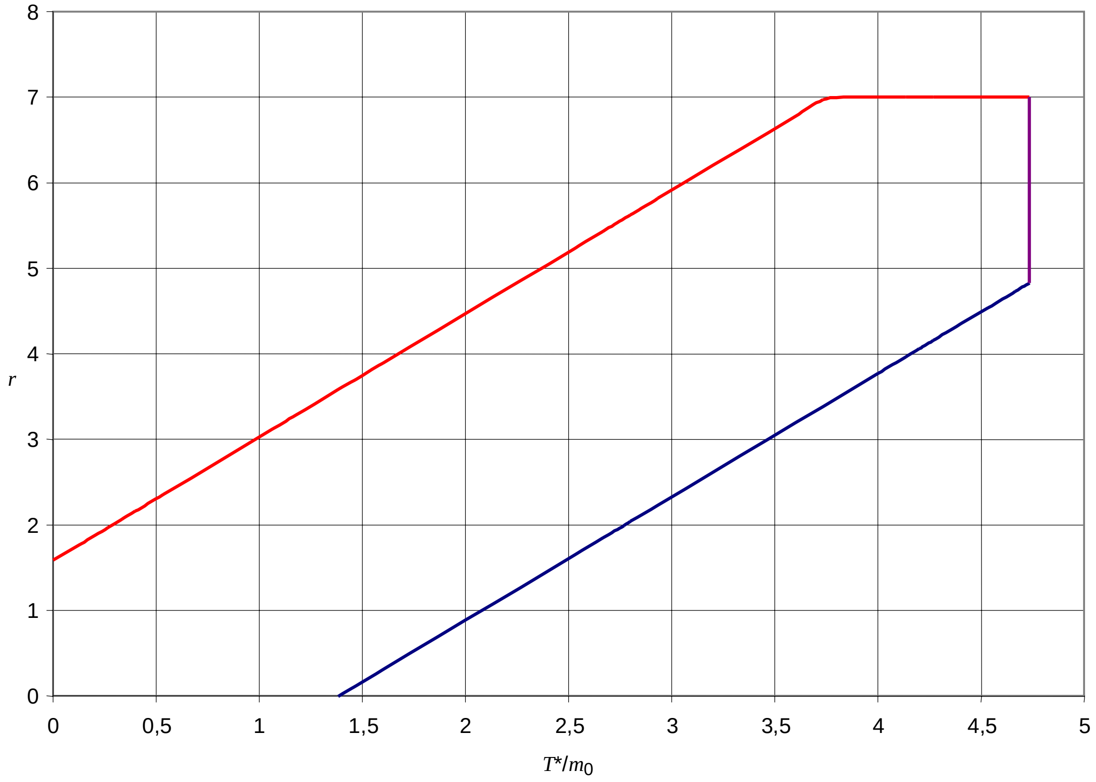

Figure A.16 – Accept and reject lines for test plan A.6

Table A.6 – Accept and reject lines for test plan A.6

r	T_r^* / m_0 **Reject** (equal or less)	T_a^* / m_0 **Accept** (equal or more)
0	–	1,39
1	–	2,08
2	0,288	2,77
3	0,981	3,47
4	1,67	4,16
5	2,37	4,73
6	3,06	4,73
$r_0 = 7$	3,75	4,73
8	4,73	N/A

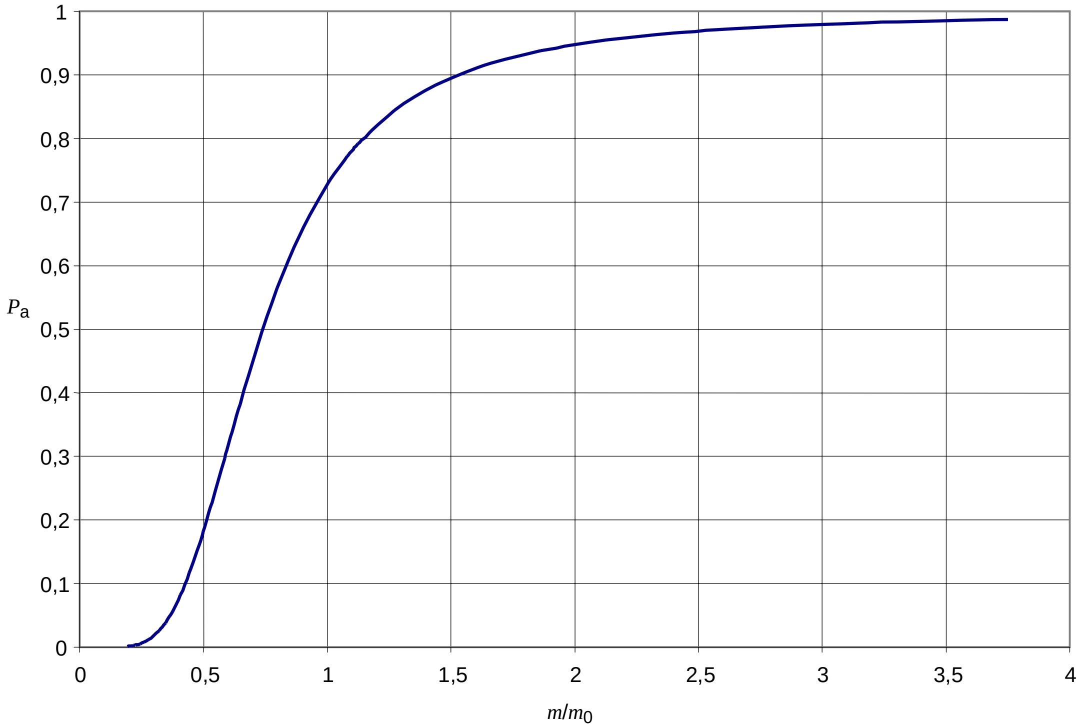

Figure A.17 – Test plan A.6 - Operating characteristic curve

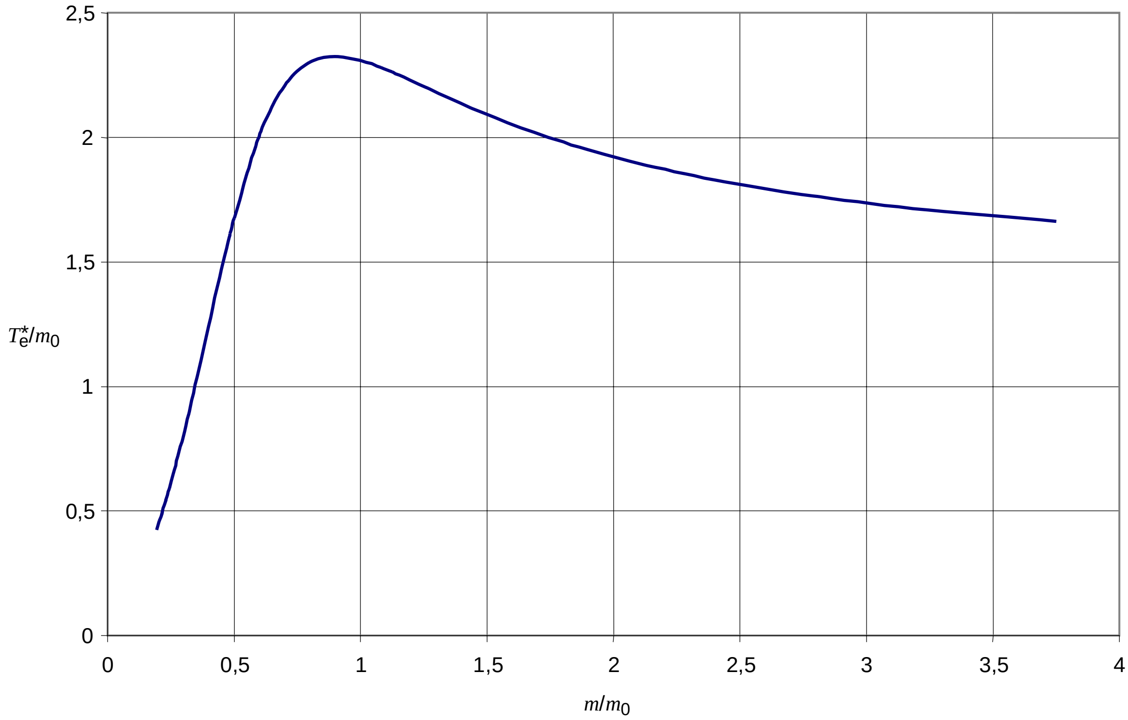

Figure A.18 – Test plan A.6 – Expected accumulated test time to decision

A.7 Test plan A.7 – α = 0,20; β = 0,20; *D* = 3,0

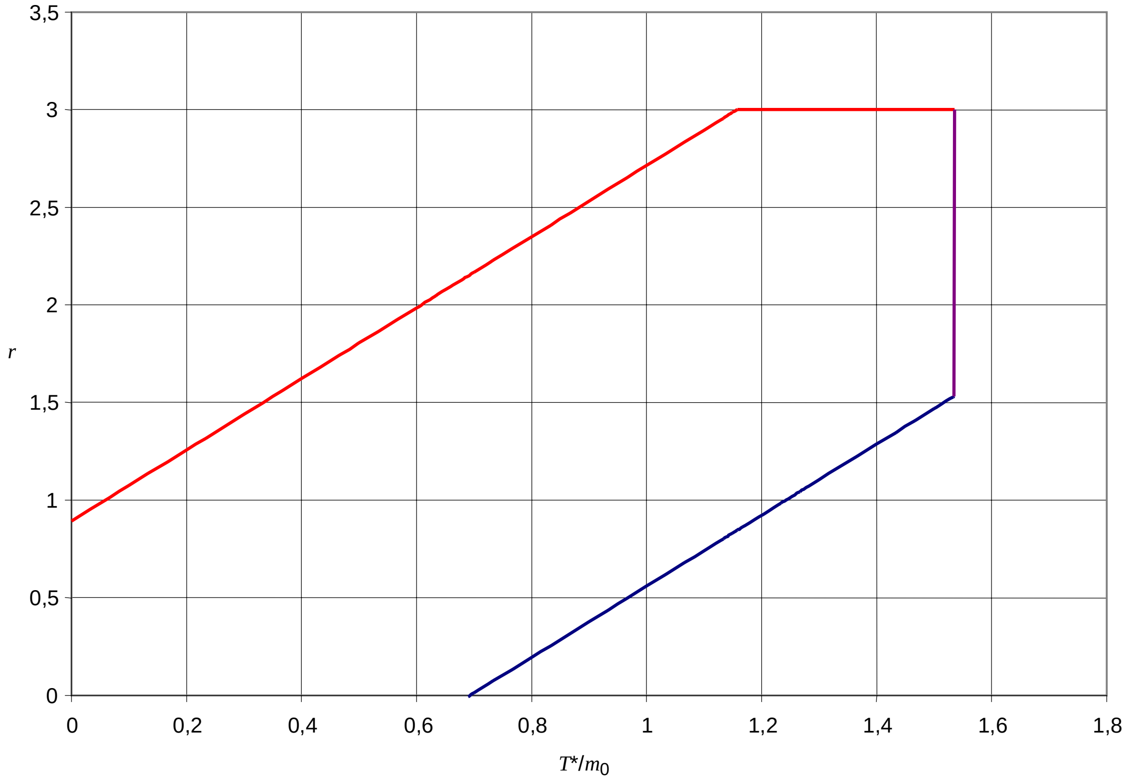

Figure A.19 – Accept and reject lines for test plan A.7

Table A.7 – Accept and reject lines for test plan A.7

r	T_r^* / m_0 **Reject** (equal or less)	T_a^* / m_0 **Accept** (equal or more)
0	-	0,693
1	0,059	1,24
2	0,608	1,54
$r_0 = 3$	1,16	1,54
4	1,54	N/A

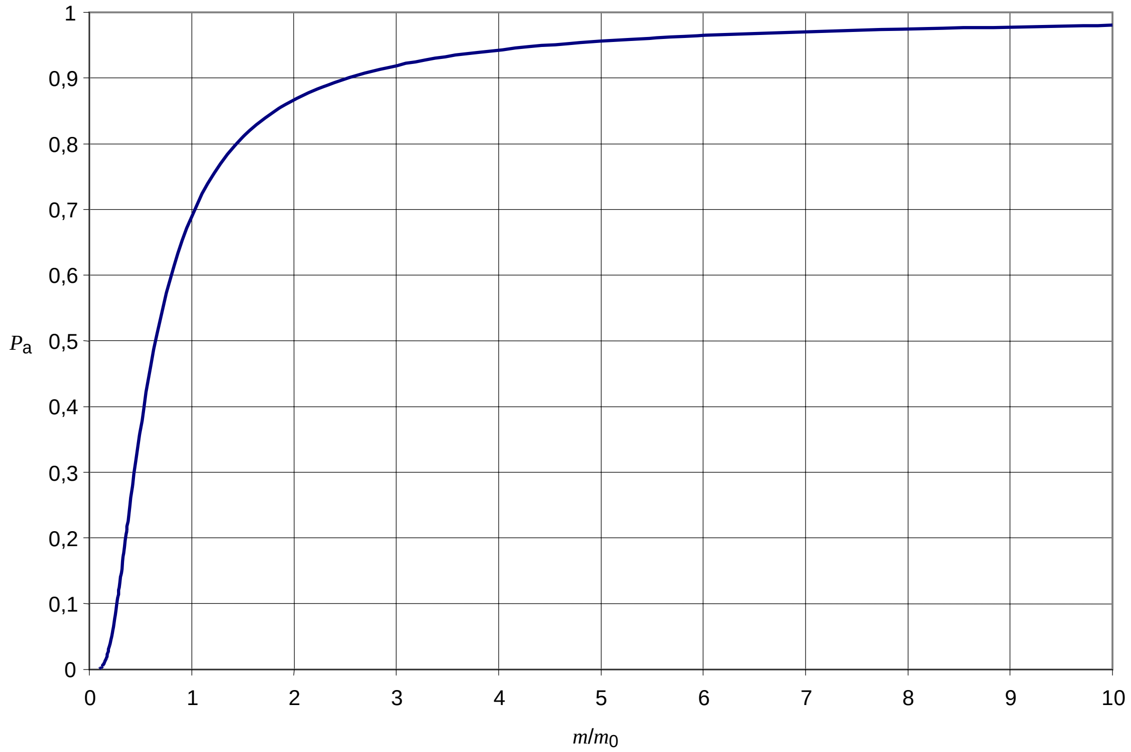

Figure A.20 – Test plan A.7 – Operating characteristic curve

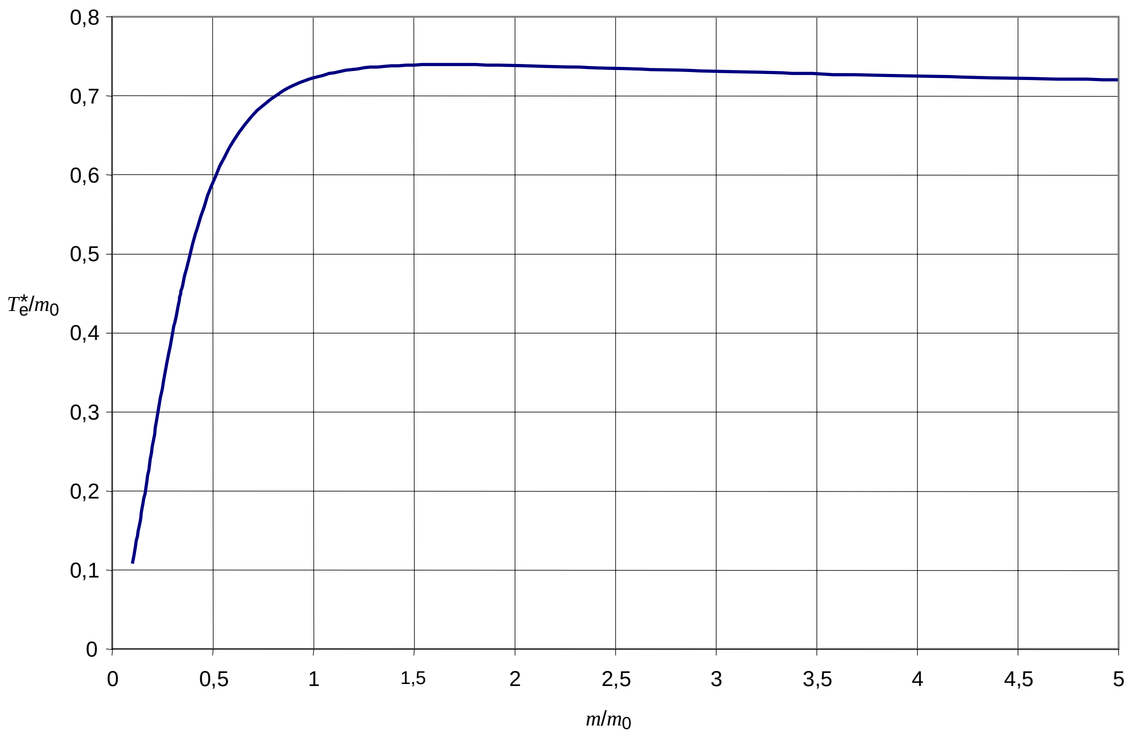

Figure A.21 – Test plan A.7 – Expected accumulated test time to decision

A.8 Test plan A.8 – α = 0,30; β = 0,30; *D* = 1,5

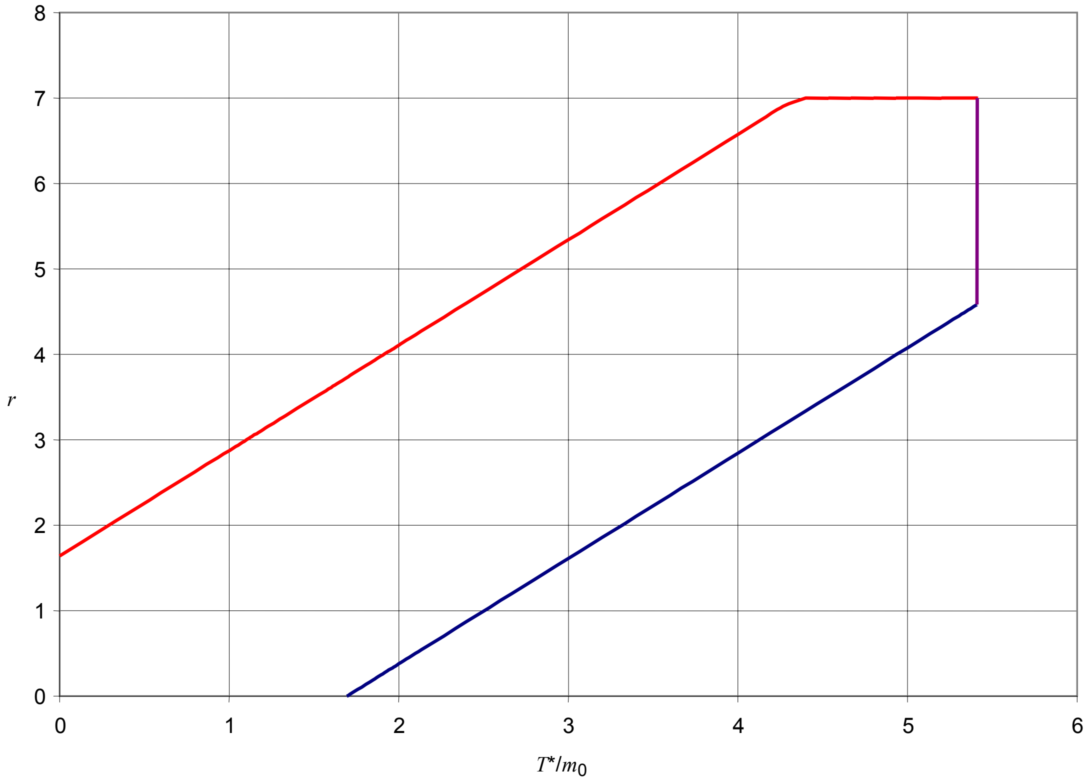

Figure A.22 – Accept and reject lines for test plan A.8

Table A.8 – Accept and reject lines for test plan A.8

r	T_r^* / m_0 **Reject** (equal or less)	T_a^* / m_0 **Accept** (equal or more)
0	–	1,69
1	–	2,51
2	0,292	3,32
3	1,10	4,13
4	1,91	4,94
5	2,72	5,41
6	3,54	5,41
$r_0 = 7$	4,35	5,41
8	5,41	N/A

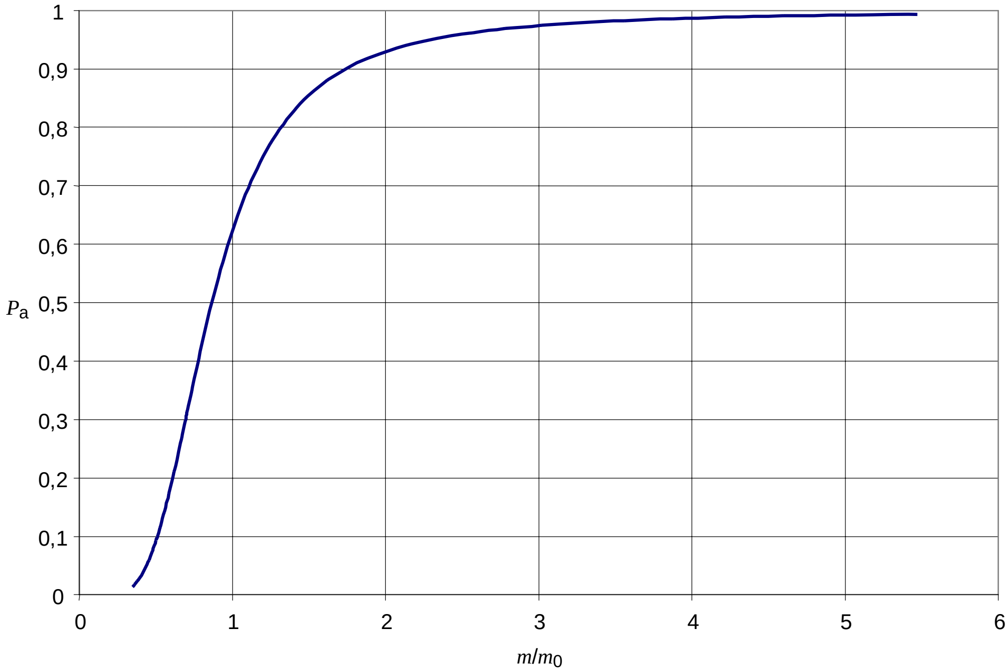

Figure A.23 – Test plan A.8 – Operating characteristic curve

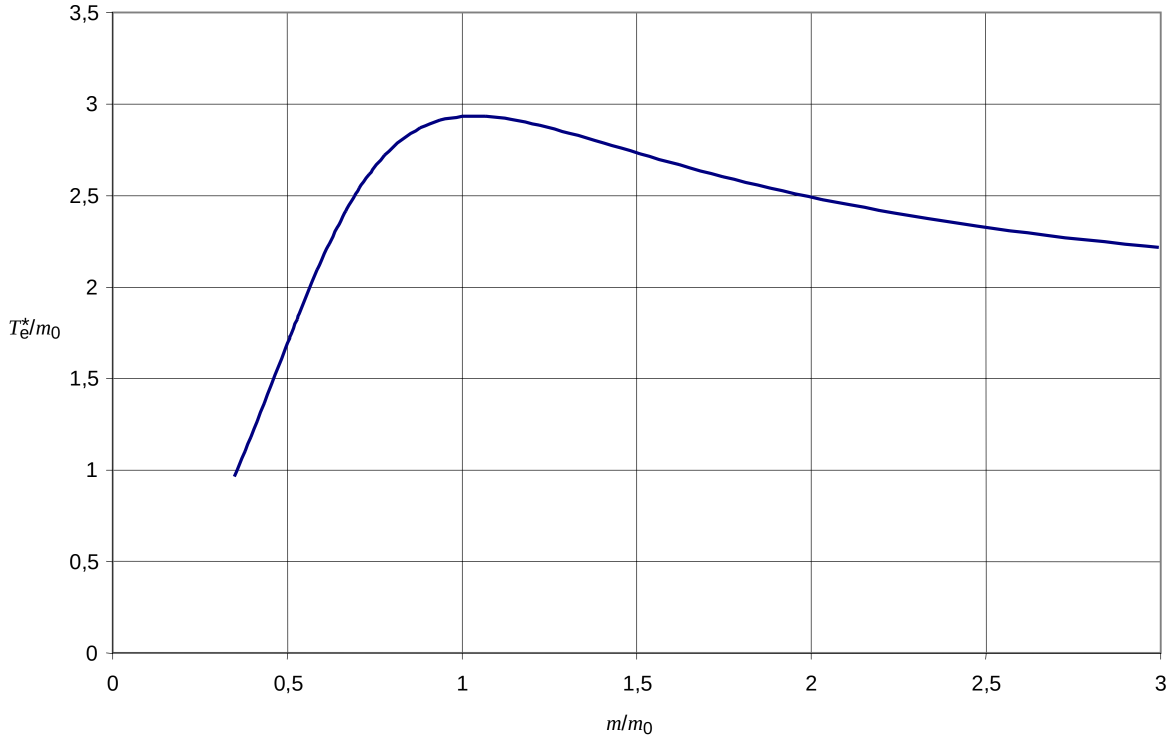

Figure A.24 – Test plan A.8 – Expected accumulated test time to decision

A.9 Test plan A.9 – α = 0,30; β = 0,30; D = 2,0

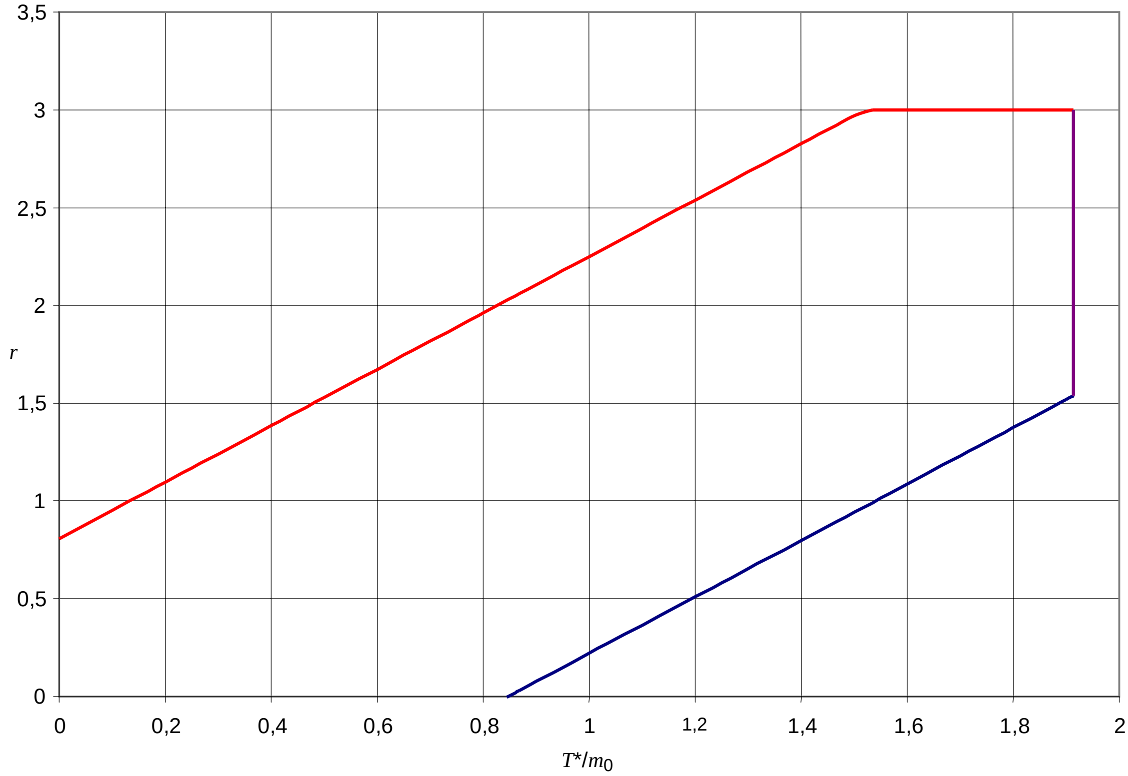

Figure A.25 – Accept and reject lines for test plan A.9

Table A.9 – Accept and reject lines for test plan A.9

r	T_r^* / m_0 **Reject** (equal or less)	T_a^* / m_0 **Accept** (equal or more)
0	–	0,847
1	0,134	1,54
2	0,827	1,91
$r_0 = 3$	1,52	1,91
4	1,91	N/A

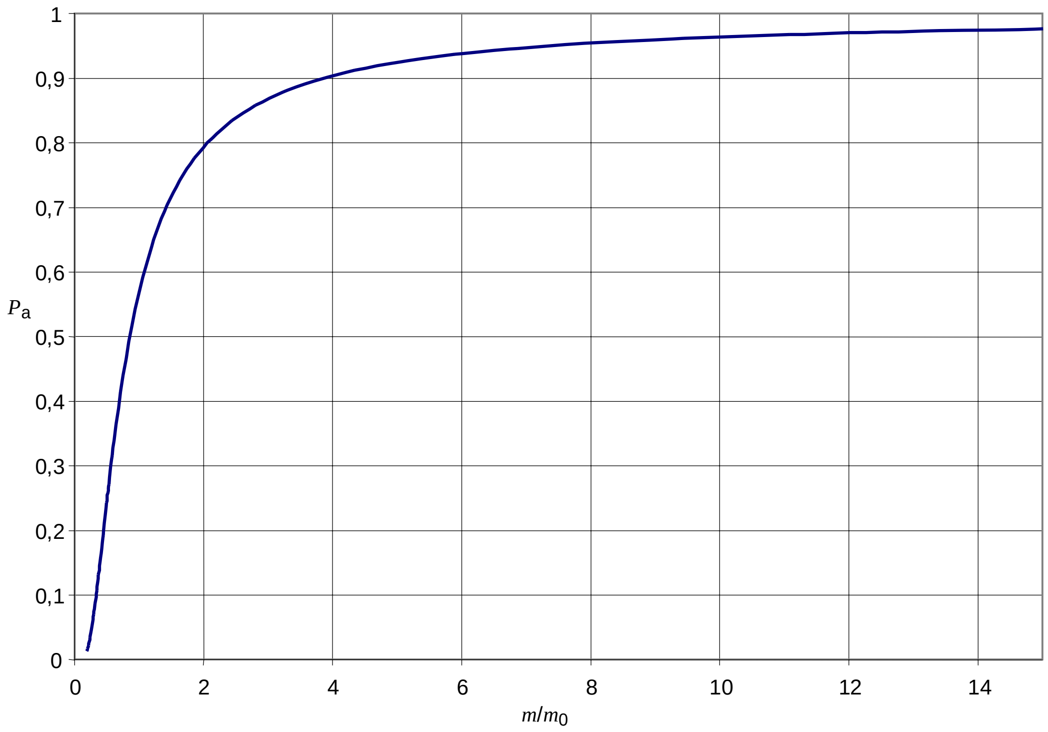

Figure A.26 – Test plan A.9 – Operating characteristic curve

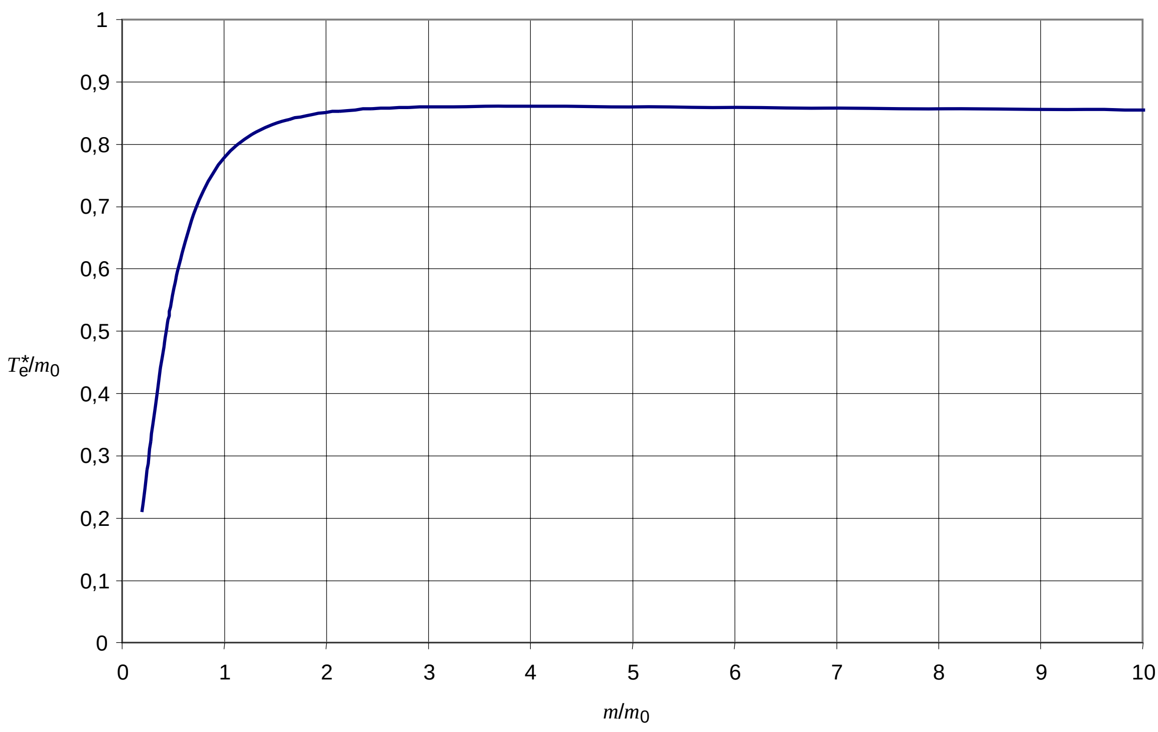

Figure A.27 – Test plan A.9 – Expected accumulated test time to decision

Annex B
(normative)

Graphs for fixed time/failure terminated test plans

NOTE 1 See Clause 7.

NOTE 2 This annex uses the symbols listed in 3.2.

NOTE 3 For the figures showing expected time to decision, the straight lines (oblique line starting at (0,0), and horizontal line) are for failure terminated test with replacement. The curved lines that approach both straight lines asymptomically are for time terminated test with replacement.

B.1 Test plans B.1 to B.4

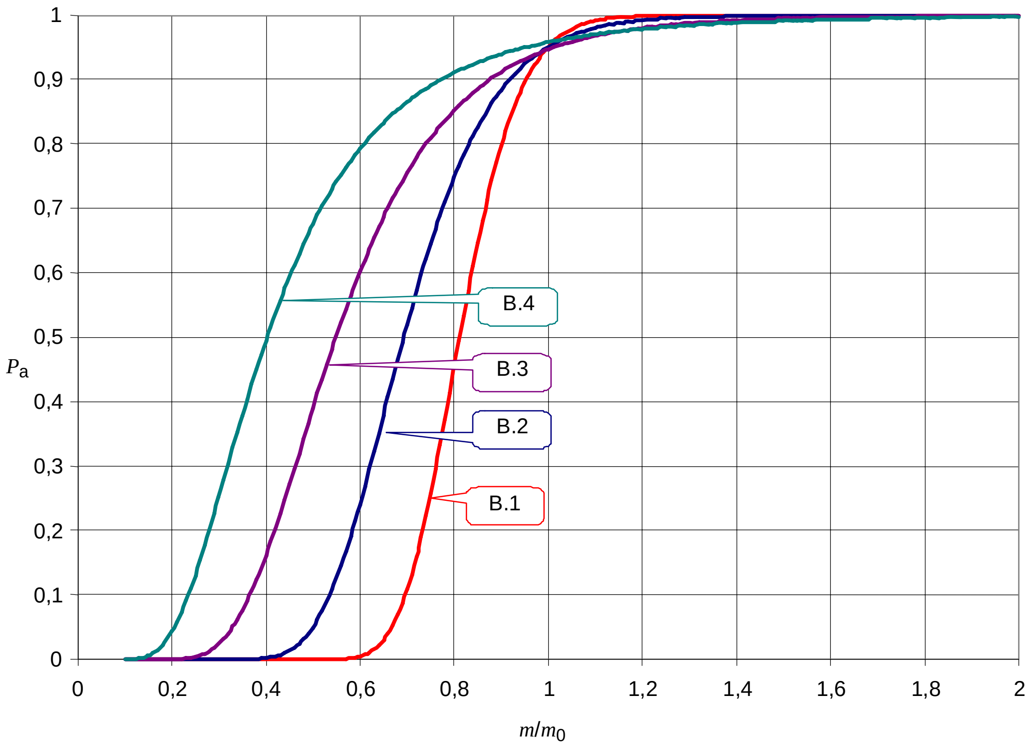

Figure B.1 – Operating characteristic curves for test plans B.1, B.2, B.3 and B.4

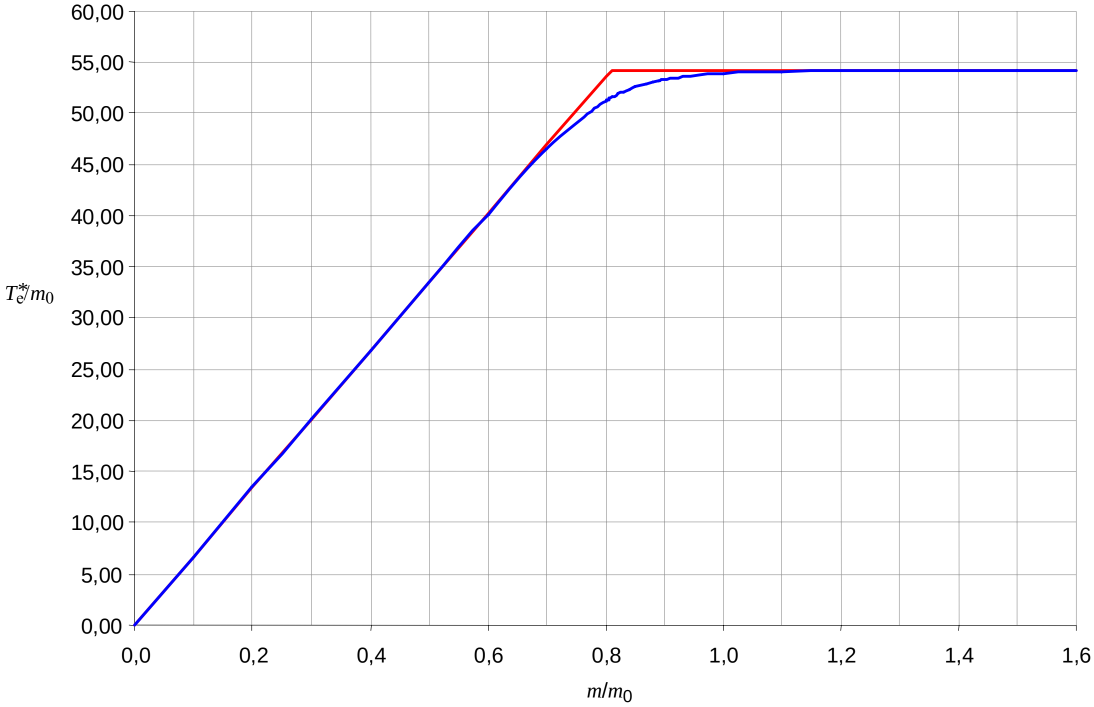

Figure B.2 – Test plan B.1 – Expected test time to decision

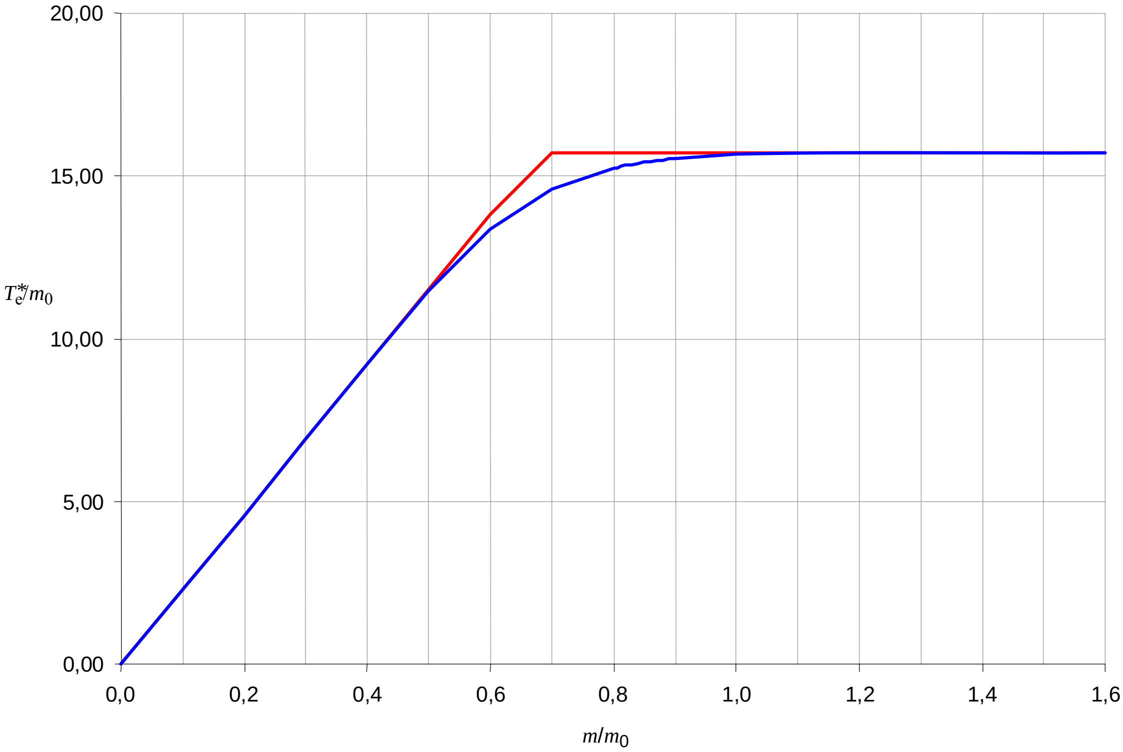

Figure B.3 – Test plan B.2 – Expected test time to decision

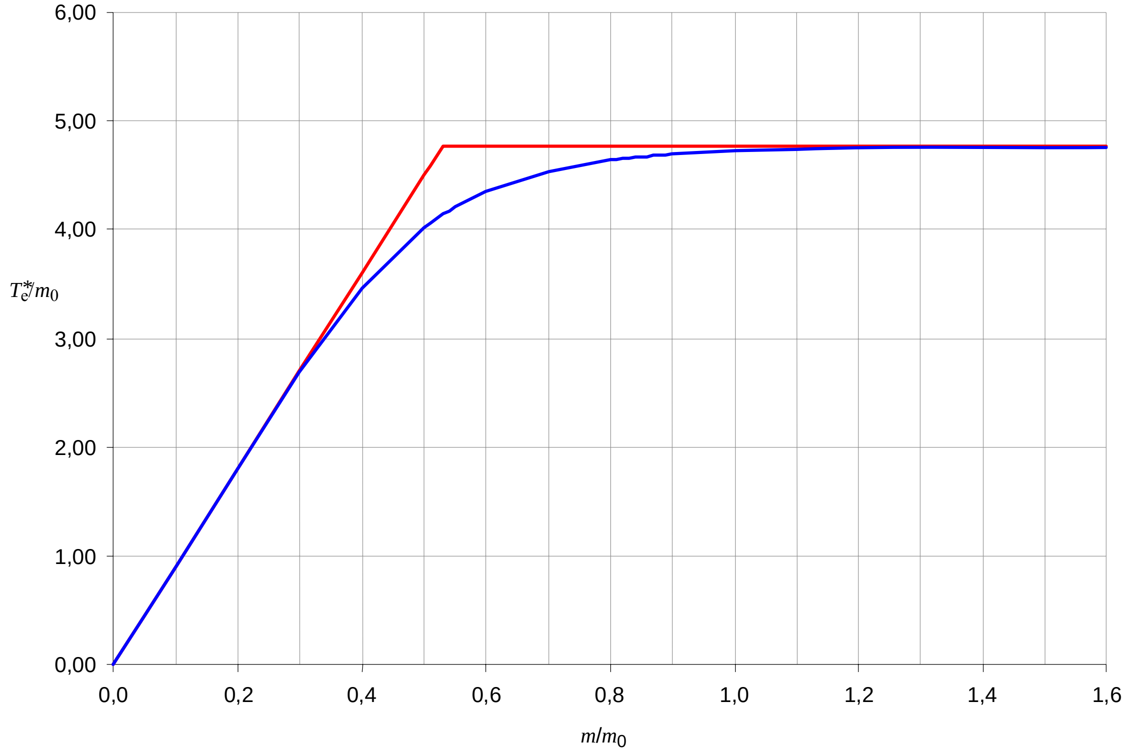

Figure B.4 – Test plan B.3 – Expected test time to decision

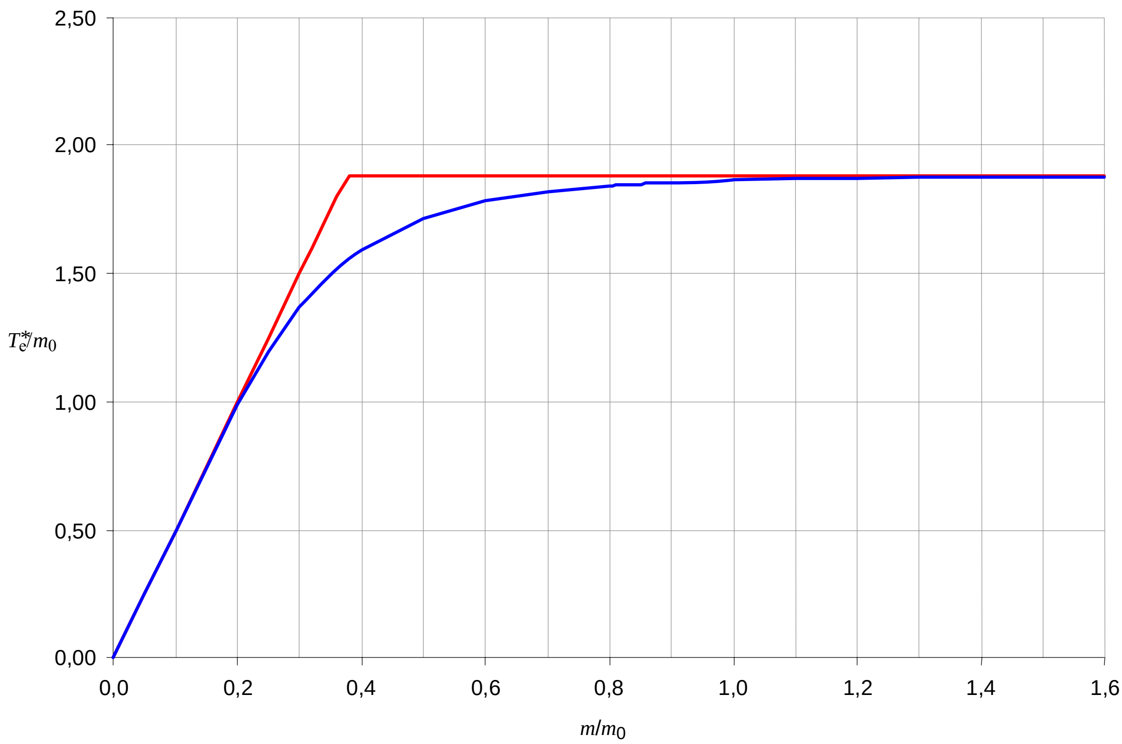

Figure B.5 – Test plan B.4 – Expected test time to decision

B.2 Test plans B.5 to B.8

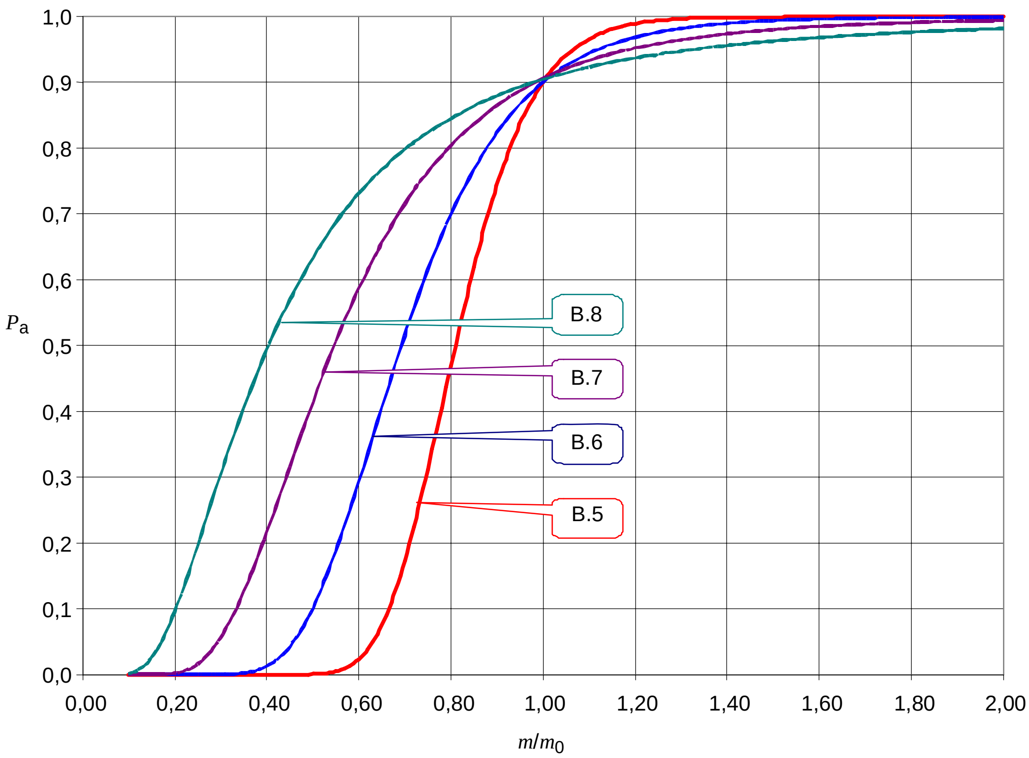

Figure B.6 – Operating characteristic curves for test plans B.5, B.6, B.7 and B.8

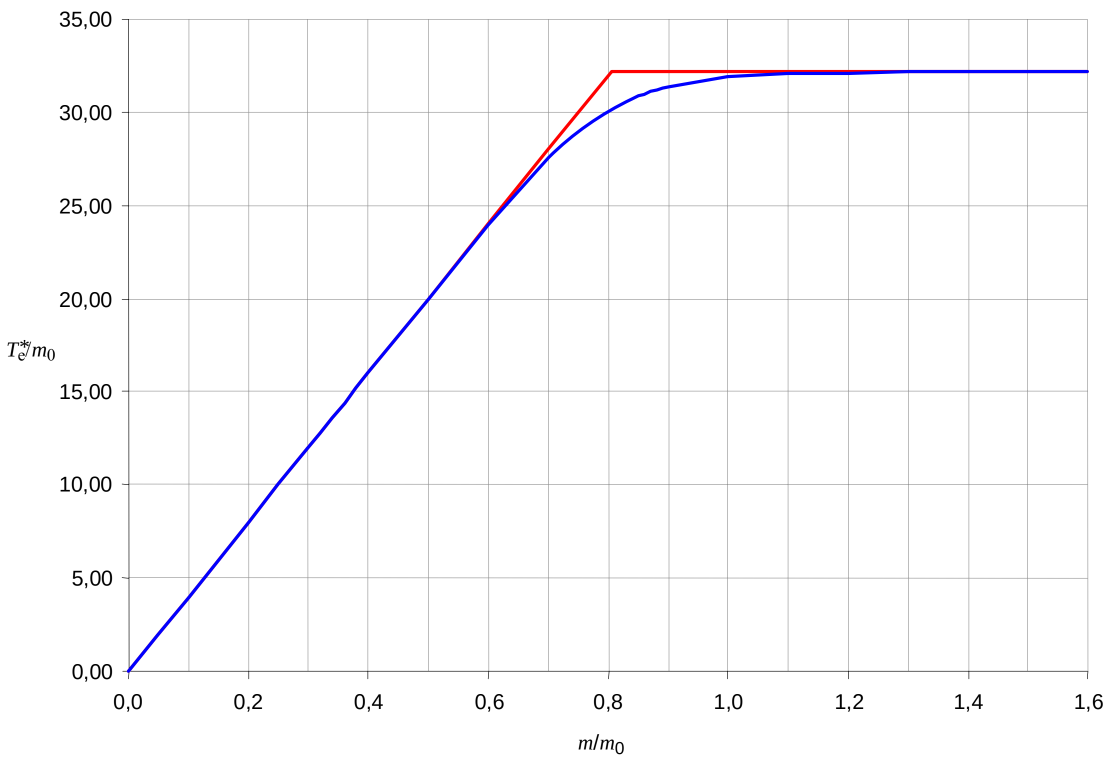

Figure B.7 – Test plan B.5 – Expected test time to decision

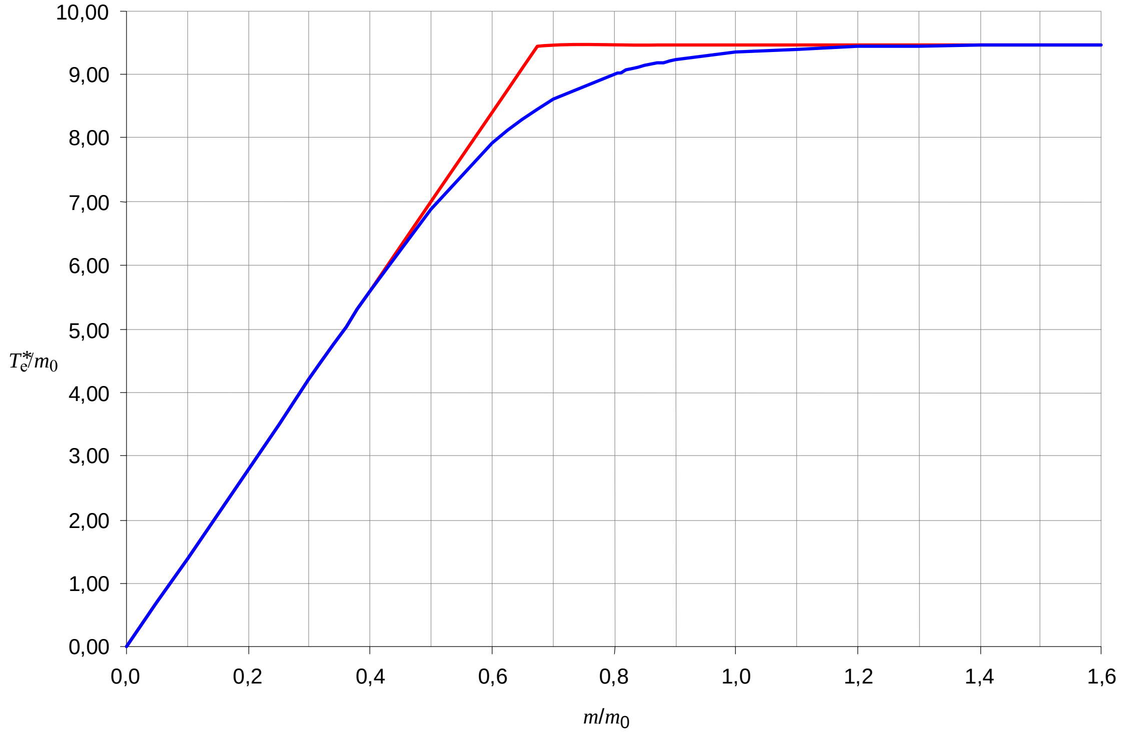

Figure B.8 – Test plan B.6 – Expected test time to decision

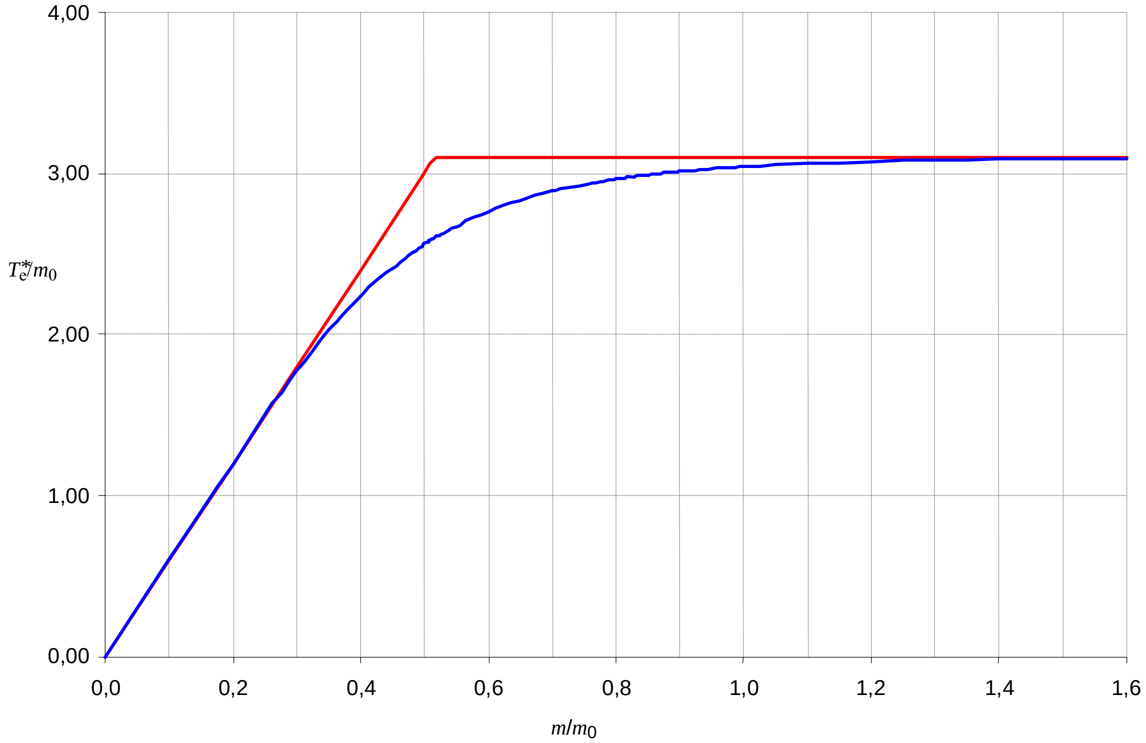

Figure B.9 – Test plan B.7 – Expected test time to decision

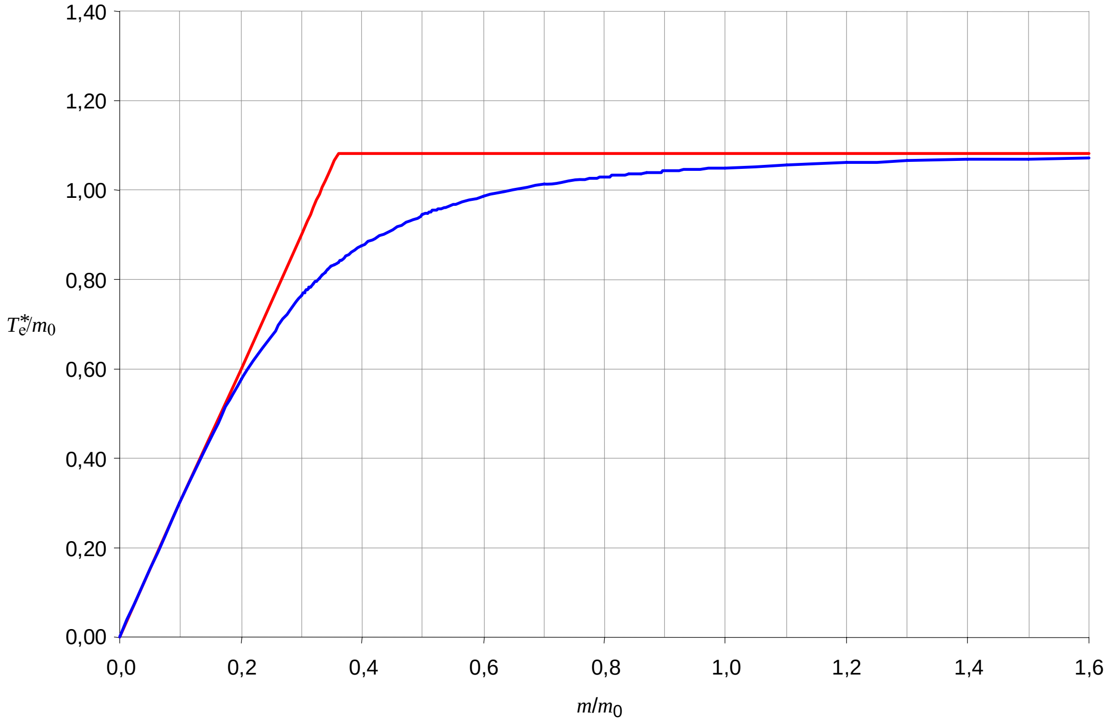

Figure B.10 – Test plan B.8 – Expected test time to decision

B.3 Test plans B.9 to B.11

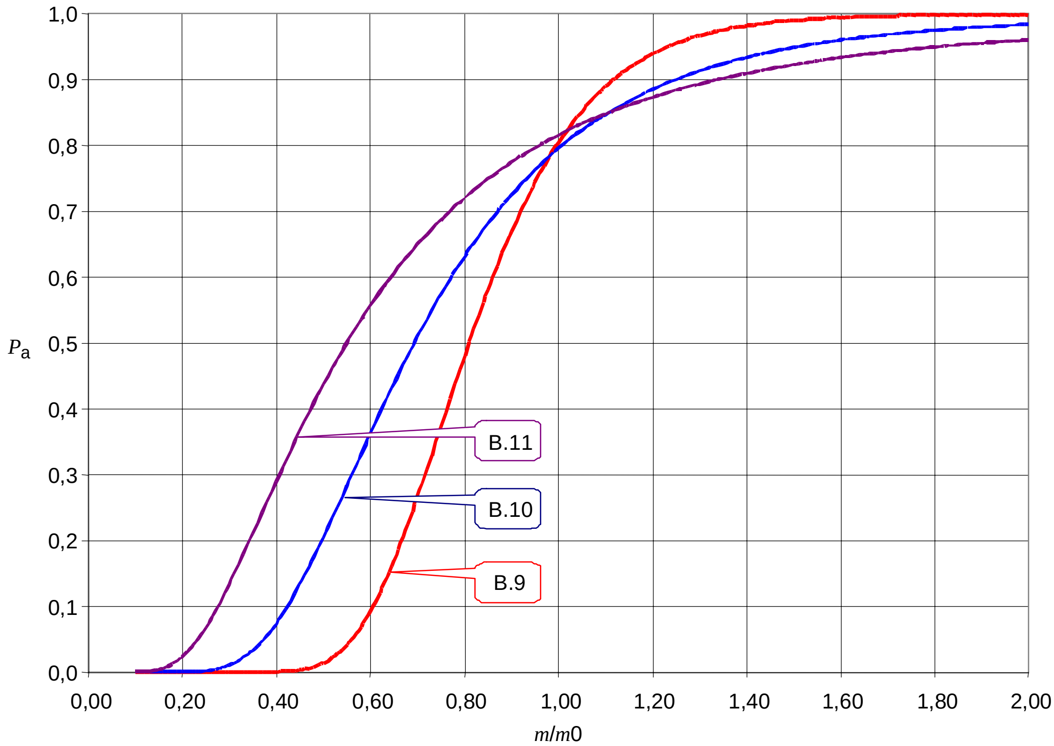

Figure B.11 – Operating characteristic curves for test plans B.9, B.10 and B.11

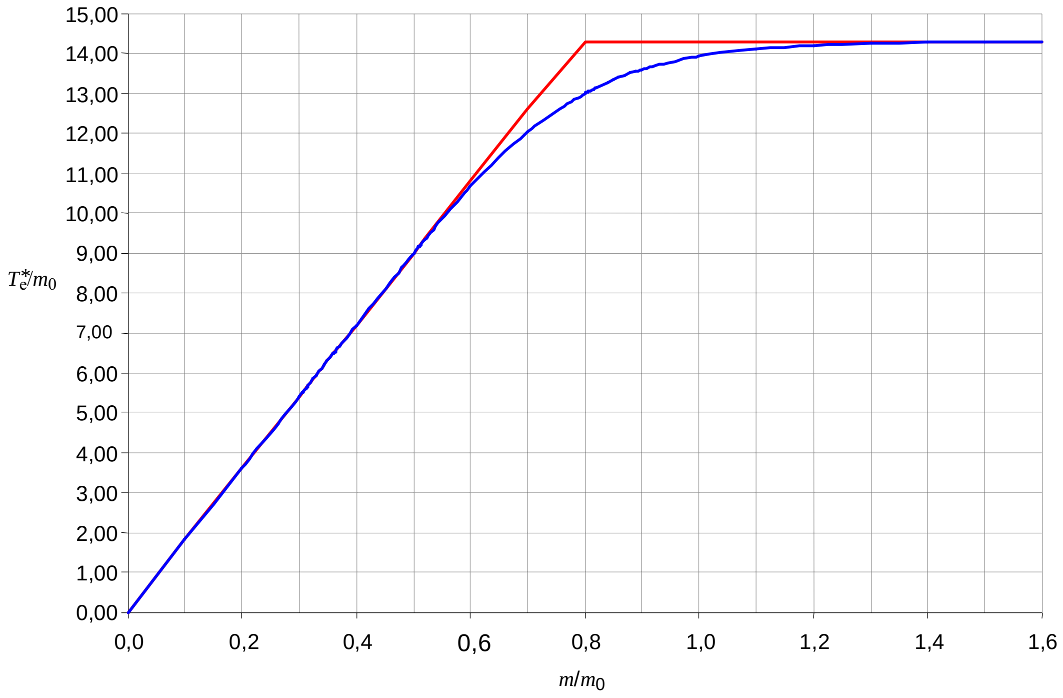

Figure B.12 – Test plan B.9 – Expected test time to decision

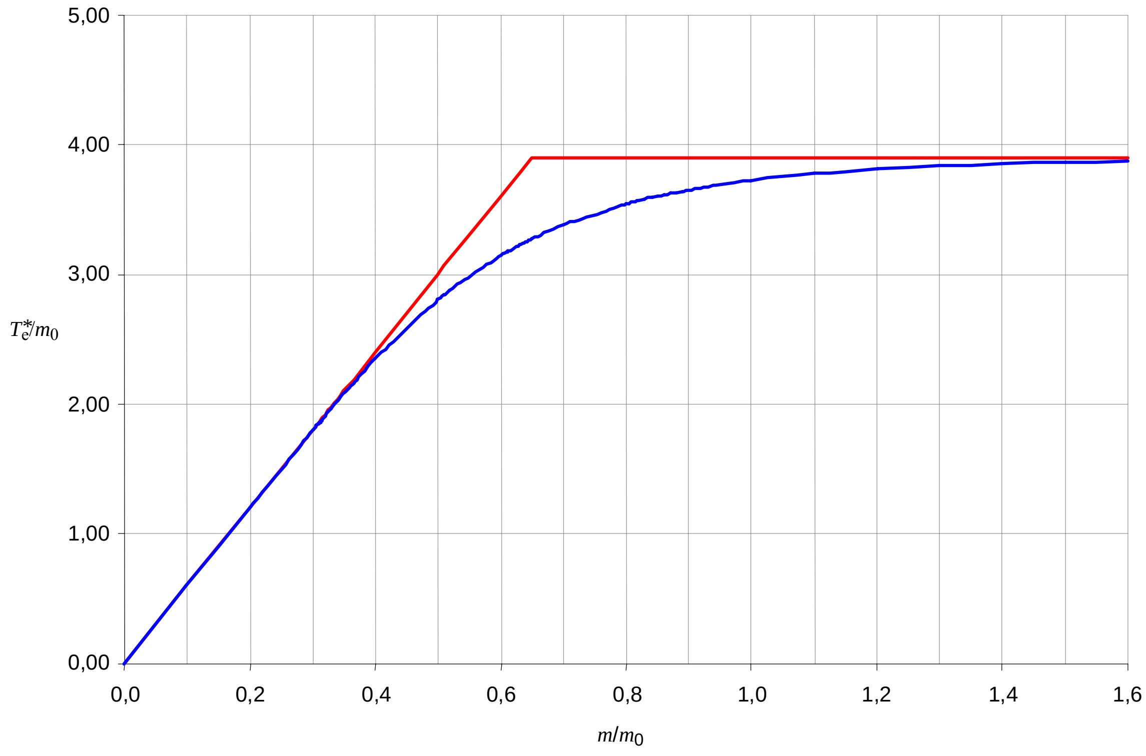

Figure B.13 – Test plan B.10 – Expected test time to decision

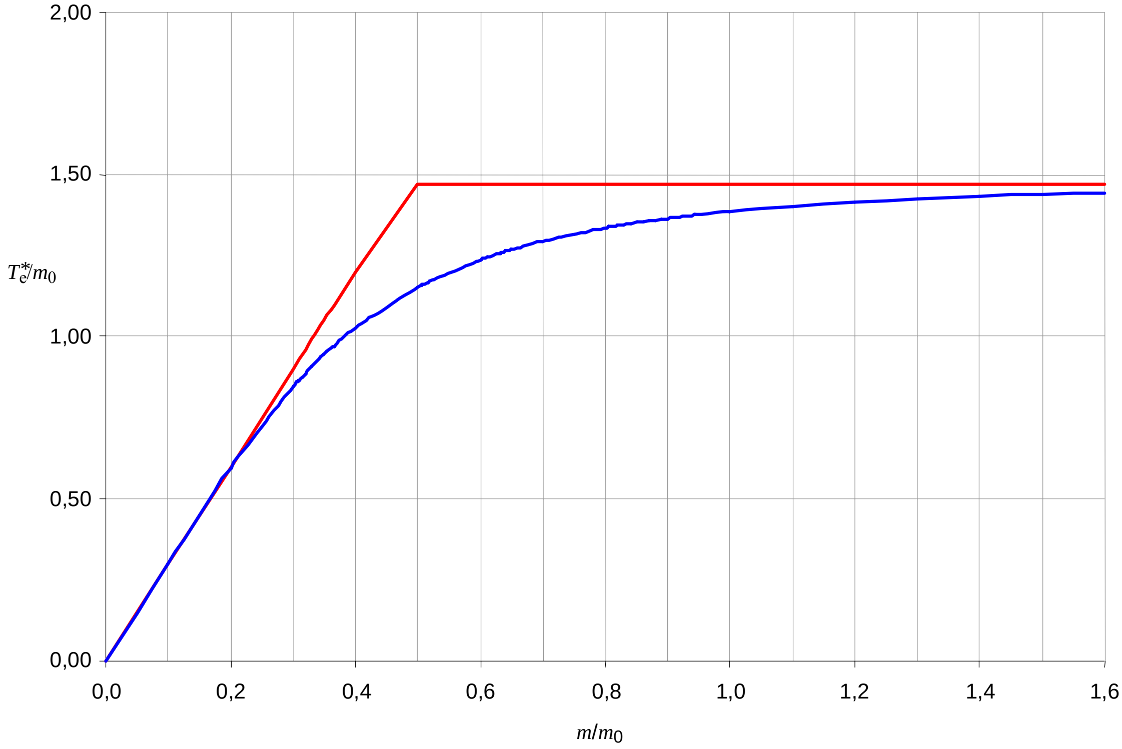

Figure B.14 – Test plan B.11– Expected test time to decision

B.4 Test plans B.12 to B.13

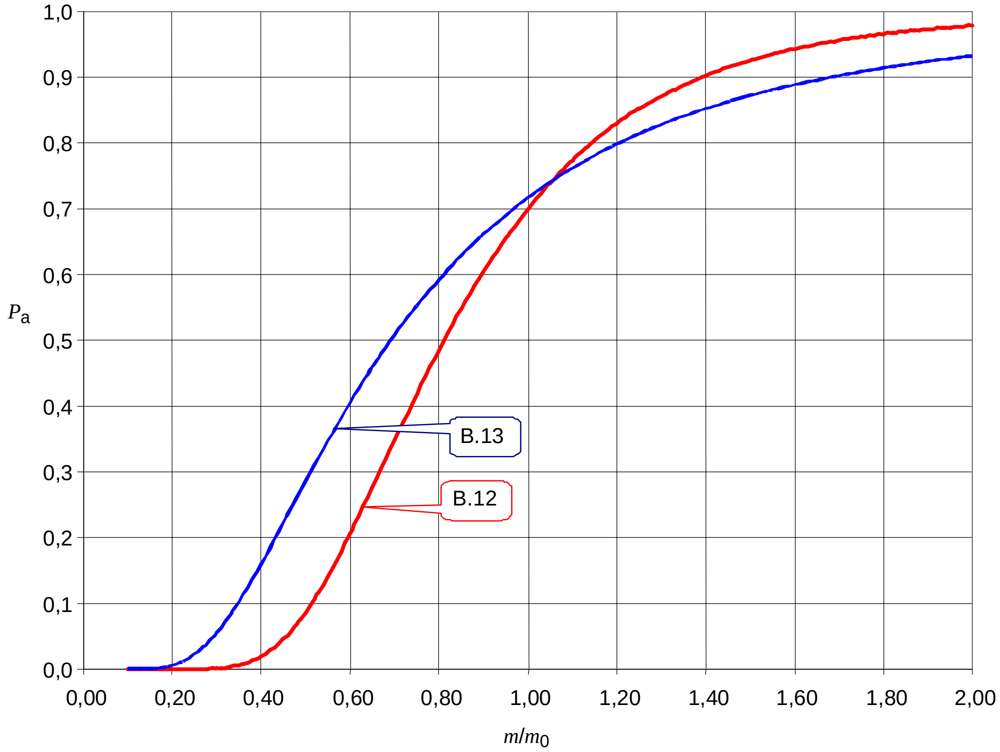

Figure B.15 – Operating characteristic curves for test plans B.12 and B.13

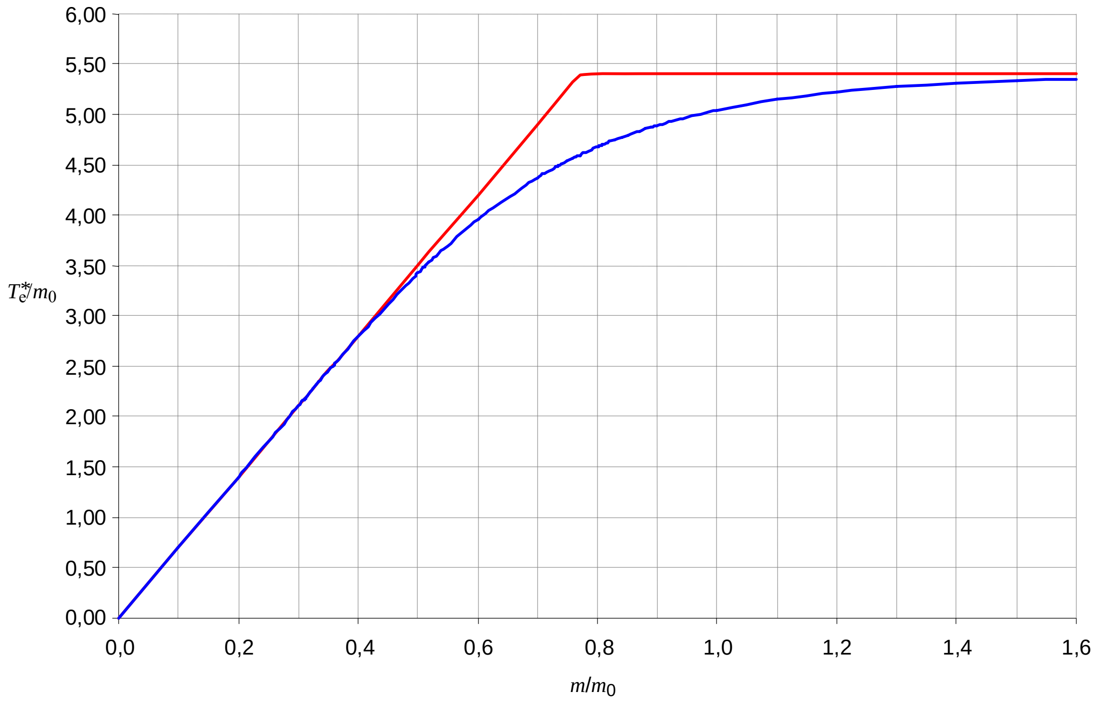

Figure B.16 – Test plan B.12 – Expected test time to decision

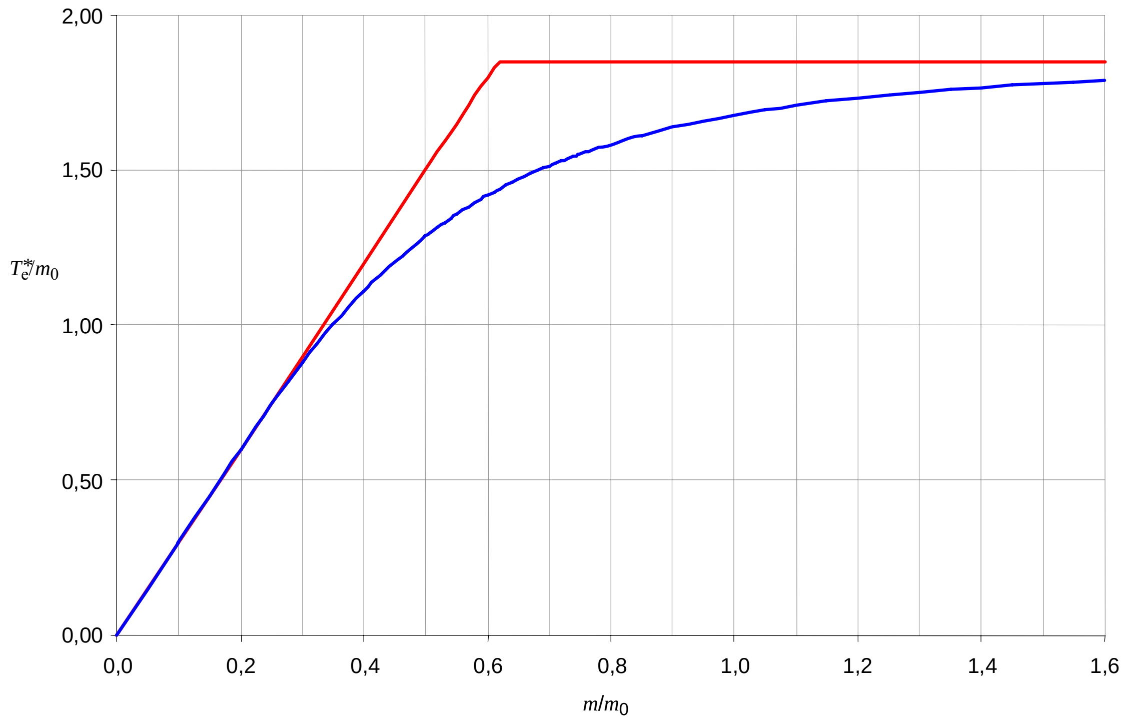

Figure B.17 – Test plan B.13 – Expected test time to decision

Annex C
(normative)

Graphs for alternative time/failure terminated test plans

NOTE 1 See Clause 8.

NOTE 2 This annex uses the symbols listed in 3.2.2.

C.1 Common case, deriving *c* and *D*

This procedure is particularly suitable for cases where the following test plan parameters are specified: test time for each item, here assumed to be the same for all items, stated as termination criterion, t_t^* ;

– number of test items, *n*;

– equal nominal producer's and consumer's risks, $\alpha = \beta$

– maximum value of the discrimination ratio, if required.

On the basis of these parameters, the unknown parameters:

– actual discrimination ratio, *D*

– acceptable number of failures, *c*

are derived, using the figures in C.4, as stated below.

If the derived actual discrimination ratio, *D* exceeds the specified maximum value, or is otherwise judged to be unsuitable, the test time shall be exceeded, or other test plan parameters modified. The calculations are then repeated and reappraised until the conditions are fulfilled.

a) General procedure

Calculate $\mu_0 = \lambda_0 t_t^* n$

NOTE If $\mu_0 < 1$, it is generally recommended to extend either test time t_t^*, or number of items *n*, in order to achieve suitable test plans.

b) Procedure for μ_0 between 1 and 5

1) In Figure C.1, find μ_0 (already calculated) on the dashed curve for the specified $\alpha = \beta$.

2) Find the nearest solid *c* curve to this point in the vertical direction, and thus determine *c.*

3) Read the discrimination ratio *D* from the *c* curve at the μ_0 value.

4) From this latter point, by interpolation between the dashed curves, the true $\alpha' = \beta'$ risks may be estimated; alternatively, the risks may be read from the corresponding operating characteristic curve in Figure C.2.

5) If the values determined for *D* and $\alpha' = \beta'$ are not acceptable, increase or decrease *c* by 1.

EXAMPLE

Specified: μ_0 = 2,7; $\alpha = \beta$ = 10 %

To be derived: *c, D*, α', β'

From Figure C.1: *c* = 4 with *D* = 2,75, and $\alpha' = \beta' \approx$ 14 %

c) Procedure for μ_0 between 5 and 500

1) Read the discrimination ratio *D* in Figure C.3 for μ_0 (already calculated) on the curve for the specified $\alpha = \beta$.
2) Read $\Delta\mu_0$ in Figure C.4 for the μ_0 calculated on the curve for the specified $\alpha = \beta$.
3) Calculate *c* by adding $\Delta\mu_0$ and μ_0 and rounding to the nearest integer, that is $c = (\Delta\mu_0 + \mu_0)$.

EXAMPLE

Specified: μ_0 = 40; $\alpha = \beta$ = 10 %

To be derived: *c*, *D*

From Figure C.3: *D* (10 %) = 1,45

From Figure C.4: *c* (10 %) = $(40 + 7{,}8)_{\text{rounded}}$ = 48

C.2 Deriving risks, $\alpha = \beta$

In addition to λ_0 the following test plan parameters shall be specified:

– test time t_t^*;
– number of test items *n* ;
– discrimination ratio *D*.

For small values of μ_0 true risks are determined; for large values of μ_0, nominal risks $\alpha = \beta$.

a) General procedure

Calculate $\mu_0 = \lambda_0 t_t^* n$ or $\mu_0 = \lambda_0 T_t^*$.

b) Procedure for μ_0 between 1 and 5

1) Find in Figure C.1 the nearest *c* curve to the point (μ_0, *D*), and thus determine *c*.
2) Read in Figure C.2, the true risks α' and β' from the corresponding operating characteristic curve for $\mu = \mu_0$ and $\mu = D\mu_0$, respectively.

EXAMPLE

Specified: μ_0 = 1 ; *D* = 3

To be derived: *c*, α', β'

From Figure C.1: *c* = 1

From Figure C.2: α' = 26,4 %, β' = 19,9 %

c) Procedure for r_0 between 5 and 500

1) Identify in Figure C.3 the point (μ_0, D) and interpolate between the $\alpha = \beta$ curves to estimate the nominal risks $\alpha = \beta$.

2) Use in Figure C.4 the nominal risks $\alpha = \beta$ to determine $\Delta\mu_0$ and then calculate as above, that is $c = (\Delta\mu_0 + \mu_0)$ $_{\text{rounded}}$.

EXAMPLE

Specified:	μ_0 = 40; D = 1,5
To be derived:	c, $\alpha = \beta$
From Figure C.3:	$\alpha = \beta$ ≈ 8 %
From Figure C.4:	$\Delta\mu_0$ = 8,7 and thus c = $(48,7)_{\text{rounded}}$ = 49

C.3 Deriving *n* or t_t^*

In addition to λ_0, one of the following sets of test plan parameters shall be specified:

a) test time t_t^*;
risks $\alpha = \beta$;
discrimination ratio D.

b) number of test items n;
risks $\alpha = \beta$;
discrimination ratio D.

Common procedure in cases a) and b):

– find D and the μ_0 value corresponding to $\alpha = \beta$ risks, from either Figure C.1, or Figure C.3;

– use the μ_0 value to determine c from either Figure C.1 or Figure C.4;

– calculate $T_t^* = \dfrac{\mu_0}{\lambda_0}$ from the μ_0 value;

– using the T_t^*, calculate either the number of test items n for specified t_t^*, $n = \dfrac{T_t^*}{t_t^*}$, or test time t_t^* for specified n, $t_t^* = \dfrac{T_t^*}{n}$

C.4 Figures

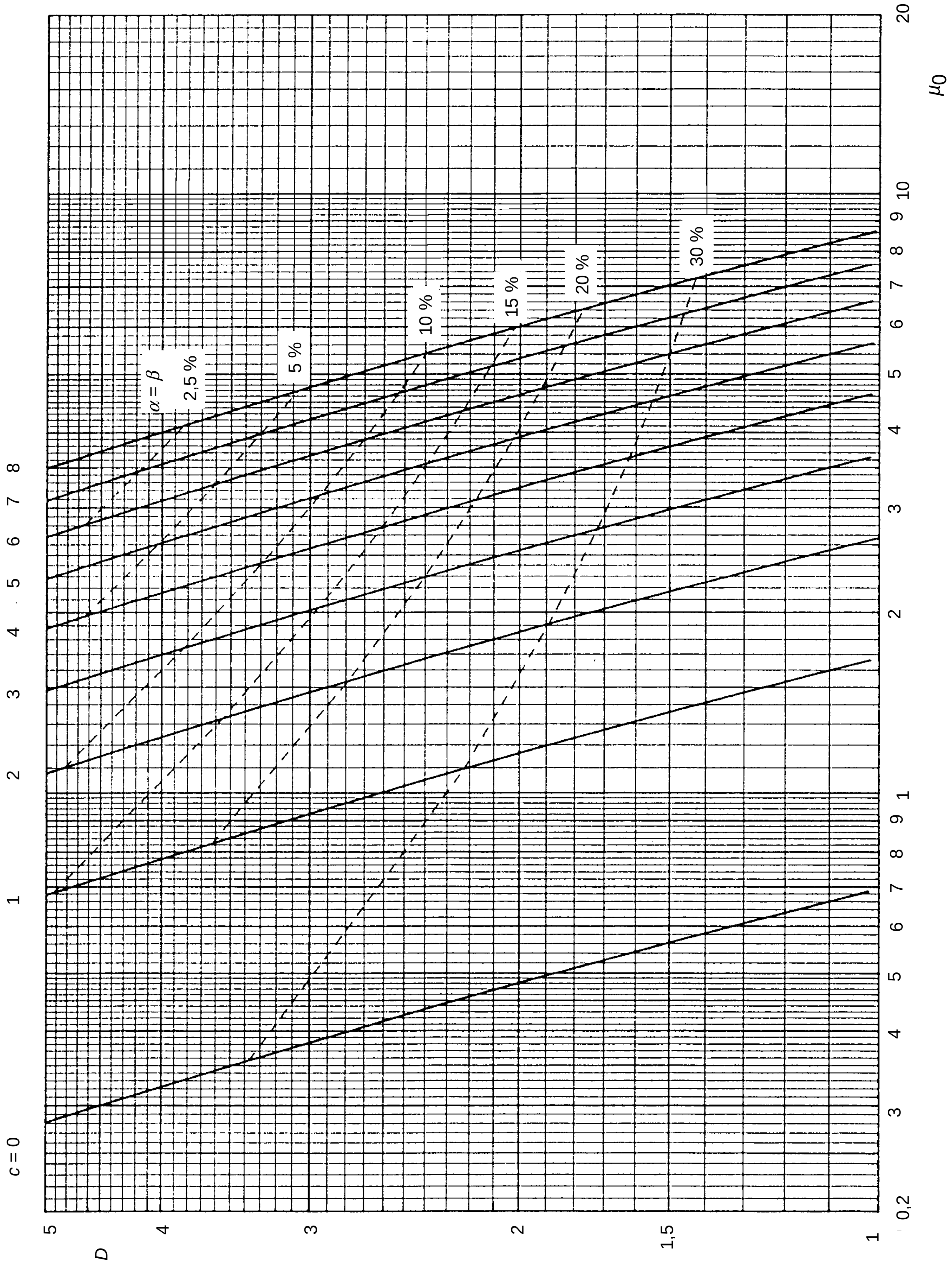

Figure C.1 – Discrimination ratio, *D*, and the acceptable number of failures, *c* = 0 to 8, as a function of the expected number of failures, μ_0 , for recommended values, 2,5%, 5%, 10%, 20%, and 30% of $\alpha = \beta$

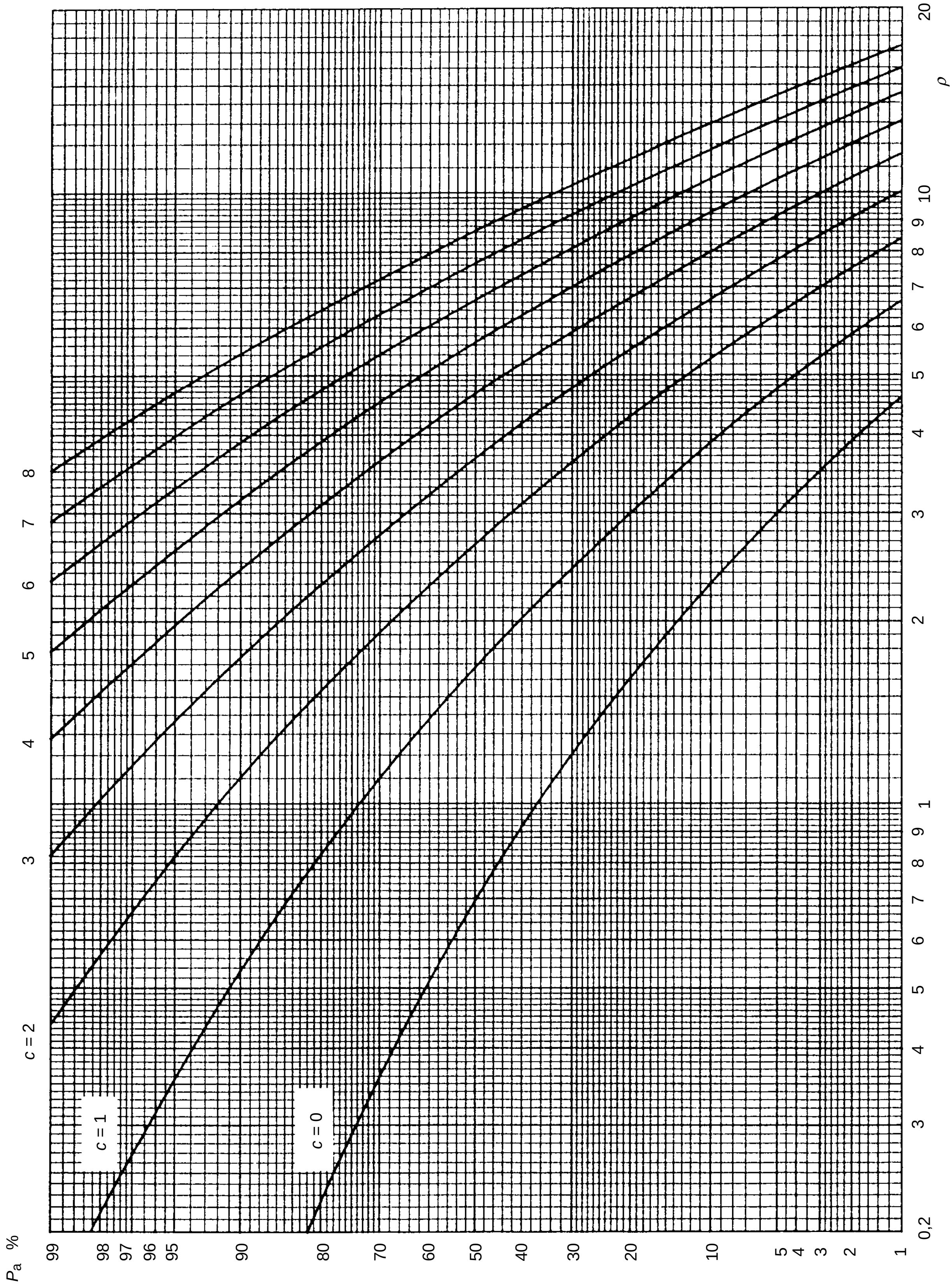

Figure C.2 – Operation characteristic curves for *c* = 0 to 8; probability of acceptance P_a as a function of the (unknown) true expected number of failures, *μ*

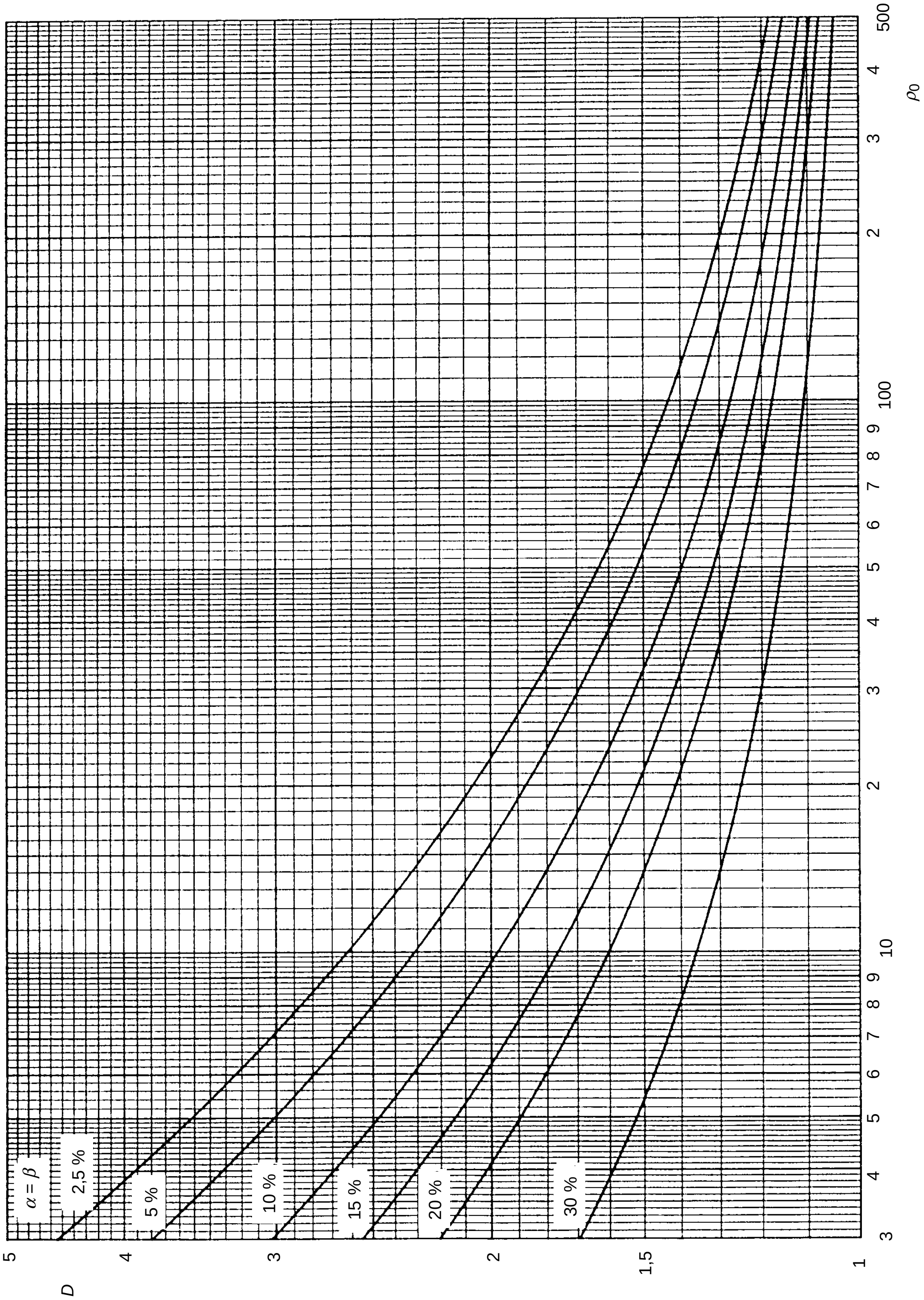

Figure C.3 – Discrimination ratio, *D*, as a function of the expected number of failures, μ_0, for recommended values, 2,5%, 5%, 10%, 15%, 20% and 30% of $\alpha = \beta$

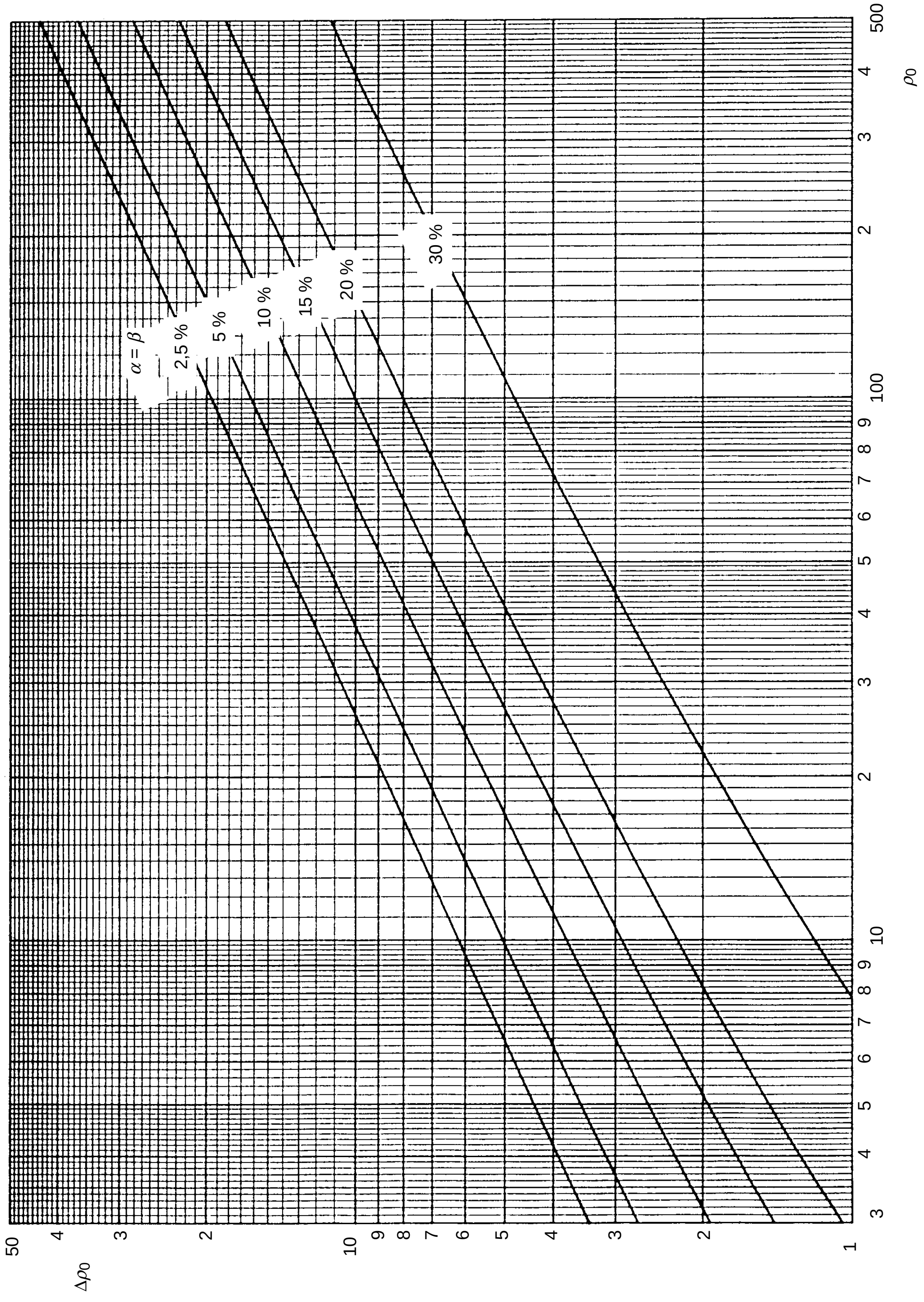

Figure C.4 – Acceptable number of failures *c* minus expected number of failures, μ_0 $\Delta\mu_0 = c - \mu_0$ as a function of the expected number of failures μ_0 for recommended values 2,5%, 5%, 10%, 20%, and 30% of $\alpha = \beta$

Annex D
(normative)

Tables and graphs for combined test plans and additional sequential test plans

NOTE 1 This annex uses the symbols listed in 3.2.2.

NOTE 2 his annex contains sequential test plans and combined test plans from GOST 27.402.

D.1 General

The combined test plans bring together the advantages of time/failure terminated test plans (test plans B, Clause 7) with the advantages of the sequential test plans (see Clause 6). They reduce the test time for highly reliable test items, while they do not reject low reliability items early. They require about the same administrative work as the sequential test plans.

More test plans of these types can be found in GOST 27.402.

Annex D contains the test plans listed in Table D.1 and D.2.

Table D.1 – Sequential test plans in Annex D

Sequential test plans C *D* = 1,7		α				
		5 %	**10 %**	**15 %**	**20 %**	**30 %**
β	5 %	C1	C2	C4		
	10 %		C3		C5	
	20 %				C6	C7
	30 %					C8

Table D.2 – Combined test plans in Annex D

Combined test plans D *D* = 1,7		α				
		5 %	**10 %**	**15 %**	**20 %**	**30 %**
β	5 %	D1	D2	D4		
	10 %		D3		D5	
	20 %				D6	D7
	30 %					D8

For each test plan, the following parameters are listed in tables and plotted:

– the accept and reject lines as the number of failures r as a function of the normalized accumulated test time T^*/m_0;

– the expected test time T_e^* as a function of the normalized accumulated test time T^*/m_0;

– the expected test time to acceptance T_e^* (+) as a function of normalized accumulated test time T^*/m_0;

– the operating characteristic P_a as a function of the normalized accumulated test time T^*/m_0.

In order to make it easy to compare the test plans of type C and D they are plotted in the same figures.

D.2 Test plans C.1 and D.1 – α = 5 %, β = 5 %; D = 1,7

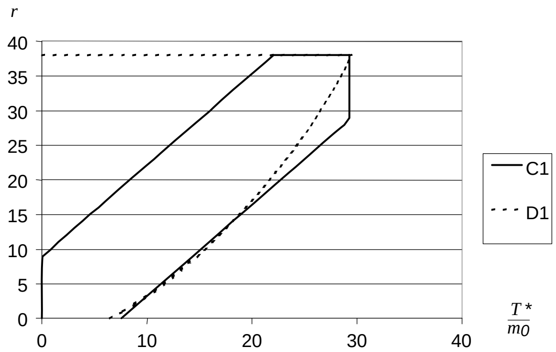

Figure D.1 – Accept and reject lines

Table D.3 – Accept and reject lines

Failures r	Accumulated test time T^* / m_0				Failures r	Accumulated test time T^* / m_0			
	Plan C1		Plan D1			Plan C1		Plan D1	
	Reject	Accept	Reject	Accept		Reject	Accept	Reject	Accept
0	0,000 0	7,584 0	No	6,454 0	21	9,161 7	23,502 8	No	22,187 8
1	0,000 0	8,342 0	No	7,769 1	22	9,919 7	24,260 9	No	22,714 3
2	0,000 0	9,100 1	No	8,877 6	23	10,677 8	25,018 9	No	23,228 6
3	0,000 0	9,858 1	No	9,875 2	24	11,435 8	25,7770	No	23,730 9
4	0,000 0	10,616 2	No	10,799 0	25	12,193 8	26,535 0	No	24,2214
5	0,000 0	11,374 2	No	11,668 3	26	12,951 9	27,293 0	No	24,699 9
6	0,000 0	12,132 2	No	12,494 5	27	13,709 9	28,051 1	No	25,166 6
7	0,000 0	12,890 3	No	13,285 2	28	14,468 0	28,809 1	No	25,621 3
8	0,000 0	13,648 3	No	14,045 7	29	15,226 0	29,300 0	No	26,064 0
9	0,065 2	14,406 4	No	14,780 0	30	15,984 0	29,300 0	No	26,494 4
10	0,823 2	15,164 4	No	15,490 8	31	16,742 1	29,300 0	No	26,912 5
11	1,581 3	15,922 4	No	16,180 6	32	17,500 1	29,300 0	No	27,317 9
12	2,339 3	16,680 5	No	16,851 0	33	18,258 2	29,300 0	No	27,710 4
13	3,097 4	17,438 5	No	17,503 6	34	19,016 2	29,300 0	No	28,089 5
14	3,855 4	18,196 6	No	18,139 5	35	19,774 2	29,300 0	No	28,455 0
15	4,613 4	18,954 8	No	18,759 7	36	20,532 3	29,300 0	No	28,806 1
16	5,371 5	19,712 6	No	19,365 0	37	21,290 3	29,300 0	No	29,142 5
17	6,129 5	20,470 7	No	19,956 1	r_0 = 38	22,048 4	29,300 0	No	29,463 2
18	6,887 6	21,228 7	No	20,533 5	39	29,300 0	N/A	Always	N/A
19	7,645 6	21,986 8	No	21,097 7	Always reject at 39 failures or more.				
20	8,403 6	22,744 8	No	21,649 0					

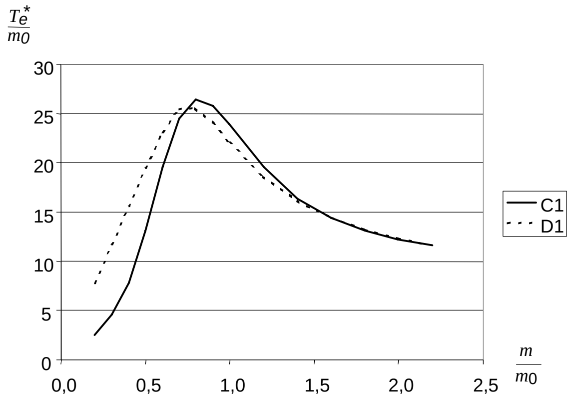

Figure D.2 – Expected test time to decision T_e^*

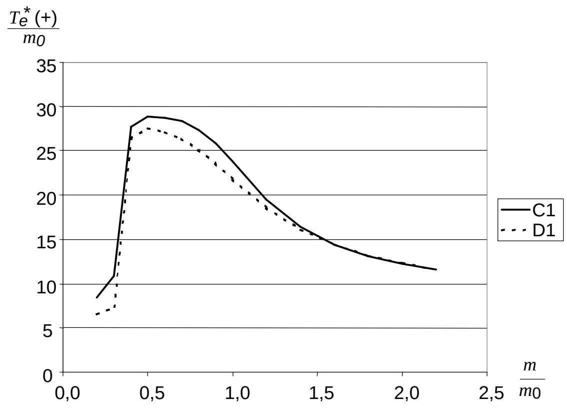

Figure D.3 – Expected test time to decision of acceptance T_e^* (+)

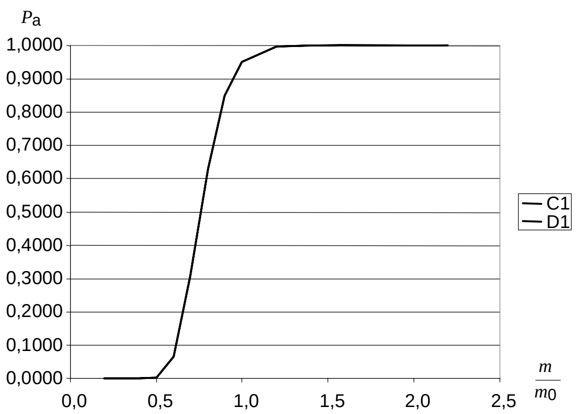

Figure D.4 – Operating characteristic P_a

Table D.4 – Expected test time to decision and operating characteristic P_a

m/m_0	Plan C1		Plan D1	
	T_e^* / m_0	$T_e^*(+) / m_0$	T_e^* / m_0	$T_e^*(+) / m_0$
0,2	2,532 8	8,371 5	7,800 0	6,521 7
0,3	4,617 3	10,888 0	11,700 0	7,395 5
0,4	7,856 3	27,663 2	15,600 0	26,378 9
0,5	13,088 4	28,835 8	19,4919	27,529 9
0,6	19,581 8	28,750 9	23,130 9	27,132 5
0,7	24,518 7	28,297 4	25,429 1	26,321 9
0,8	26,413 6	27,352 6	25,521 9	25,110 6
0,9	25,803 0	25,803 4	24,045 0	23,544 0
1,0	23,890 9	23,763 9	22,065 4	21,791 0
1,2	19,540 9	19,512 2	18,568 9	18,532 2
1,4	16,383 8	16,381 1	16,132 9	16,129 7
1,6	14,388 4	14,388 2	14,449 0	14,448 7
1,8	13,097 7	13,097 7	13,239 0	13,239 0
2,0	12,212 3	12,212 3	12,335 6	12,335 5
2,2	11,570 8	11,570 8	11,638 4	11,638 4

m/m_0	P_a	
	Plan C1	Plan D1
0,2	0,000 0	0,000 0
0,3	0,000 0	0,000 0
0,4	0,000 0	0,000 0
0,5	0,002 7	0,002 8
0,6	0,065 4	0,065 3
0,7	0,308 1	0,307 7
0,8	0,630 1	0,629 6
0,9	0,850 2	0,849 9
1,0	0,950 0	0,950 0
1,2	0,995 9	0,996 0
1,4	0,999 7	0,999 7
1,6	1,000 0	1,000 0
1,8	1,000 0	1,000 0
2,0	1,000 0	1,000 0
2,2	1,000 0	1,000 0

D.3 Test plans C.2 and D.2 – α = 10 %, β = 5 %; D = 1,7

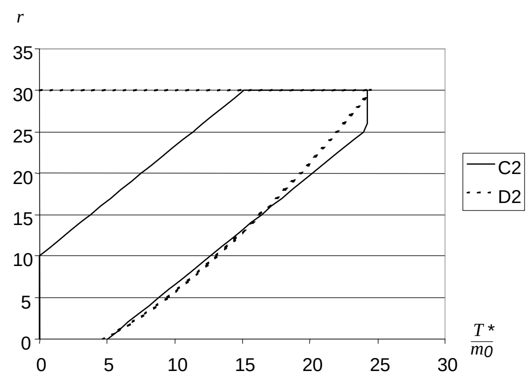

Figure D.5 – Accept and reject lines

Table D.5 – Accept and reject lines

Failures r	Accumulated test time T^*/m_0			
	Plan C2		Plan D2	
	Reject	Accept	Reject	Accept
0	0,000 0	5,032 9	No	4,721 9
1	0,000 0	5,790 9	No	5,895 8
2	0,000 0	6,549 0	No	6,899 3
3	0,000 0	7,307 0	No	7,812 9
4	0,000 0	8,065 1	No	8,667 4
5	0,000 0	8,823 1	No	9,478 7
6	0,000 0	9,581 1	No	10,256 1
7	0,000 0	10,339 2	No	11,005 8
8	0,000 0	11,097 2	No	11,732 2
9	0,000 0	11,855 3	No	12,438 3
10	0,000 0	12,613 3	No	13,126 6
11	0,720 1	13,371 3	No	13,798 8
12	1,478 2	14,129 4	No	14,456 3
13	2,236 2	14,887 4	No	15,100 5
14	2,994 3	15,645 5	No	15,732 1
15	3,752 3	16,403 5	No	16,352 1
16	4,510 3	17,161 5	No	16,961 0
17	5,268 4	17,919 6	No	17,559 4
18	6,026 4	18,677 6	No	18,147 8
19	6,784 5	19,435 7	No	18,726 5
20	7,542 5	20,193 7	No	19,295 9
21	8,300 5	20,951 7	No	19,856 3
22	9,058 6	21,709 8	No	20,407 8
23	9,816 6	22,467 8	No	20,950 6
24	10,574 7	23,225 9	No	21,485 0
25	11,332 7	23,983 9	No	22,011 0
26	12,090 7	24,240 0	No	22,528 7
27	12,848 8	24,240 0	No	23,038 1
28	13,606 8	24,240 0	No	23,539 3
29	14,364 9	24,240 0	No	24,032 3
$r_0 = 30$	15,122 9	24,240 0	No	24,517 0
31	24,240 0	N/A	Always	N/A
	Always reject at 31 failure or more.			

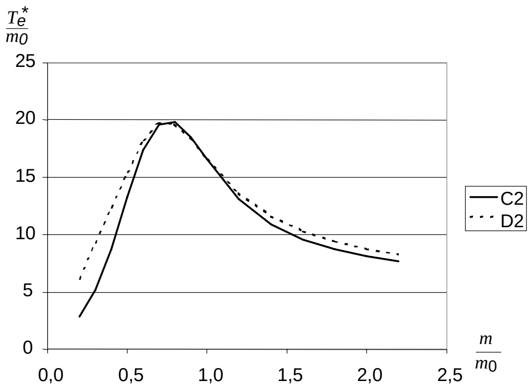

Figure D.6 – Expected test time to decision T_e^*

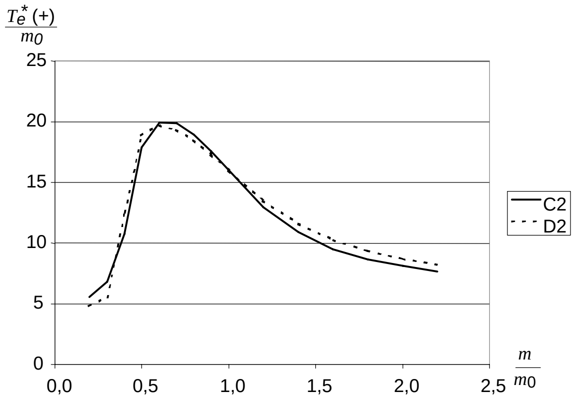

Figure D.7 – Expected test time to decision of acceptance $T_e^*(+)$

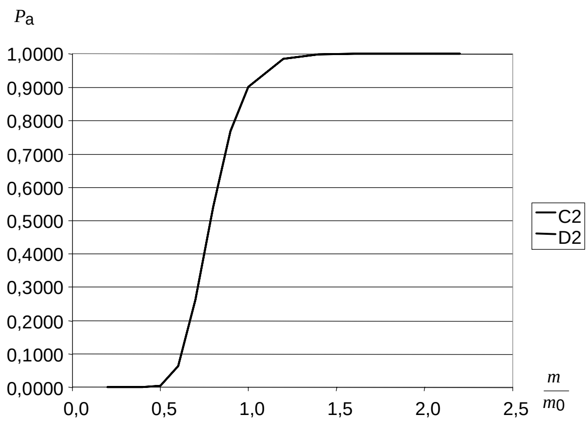

Figure D.8 – Operating characteristic P_a

Table D.6 – Expected test time to decision and operating characteristic P_a

	Plan C2		Plan D2				P_a	
m/m_0	T_e^* / m_0	$T_e^*(+) / m_0$	T_e^* / m_0	$T_e^*(+) / m_0$		m/m_0	Plan C2	Plan D2
0,2	2,841 4	5,555 5	6,200 0	4,815 2		0,2	0,000 0	0,000 0
0,3	5,181 1	6,828 7	9,300 0	5,559 4		0,3	0,000 0	0,000 0
0,4	8,722 1	10,738 8	12,399 6	12,548 4		0,4	0,000 1	0,000 0
0,5	13,311 2	17,895 5	15,475 8	18,847 2		0,5	0,004 6	0,004 4
0,6	17,370 8	19,933 3	18,230 3	19,709 2		0,6	0,063 2	0,063 3
0,7	19,643 2	19,857 7	19,790 5	19,350 6		0,7	0,261 5	0,262 5
0,8	19,816 8	18,948 8	19,643 2	18,478 7		0,8	0,541 4	0,542 3
0,9	18,491 3	17,571 7	18,306 8	17,300 3		0,9	0,769 0	0,769 3
1,0	16,576 0	15,975 4	16,586 0	15,977 0		1,0	0,900 0	0,900 0
1,2	13,116 9	12,980 5	13,602 0	13,471 0		1,2	0,985 6	0,985 5
1,4	10,896 7	10,875 5	11,615 2	11,595 0		1,4	0,998 2	0,998 2
1,6	9,551 8	9,548 9	10,307 9	10,305 1		1,6	0,999 8	0,999 8
1,8	8,692 3	8,691 9	9,402 6	9,402 2		1,8	1,000 0	1,000 0
2,0	8,104 4	8,104 3	8,743 7	8,743 6		2,0	1,000 0	1,000 0
2,2	7,678 6	7,678 6	8,244 3	8,244 3		2,2	1,000 0	1,000 0

D.4 Test plans C.3 and D.3 – α = 10 %, β = 10 %; *D* = 1,7

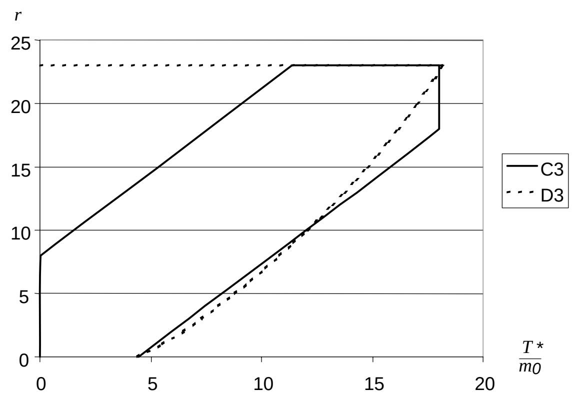

Figure D.9 – Accept and reject lines

Table D.7 – Accept and reject lines

Failures r	Accumulated test time T^*/m_0			
	Plan C3		Plan D3	
	Reject	Accept	Reject	Accept
0	0,000 0	4,429 4	no	4,426 4
1	0,000 0	5,187 4	No	5,531 2
2	0,000 0	5,945 5	No	6,466 4
3	0,000 0	6,703 5	No	7,310 1
4	0,000 0	7,461 6	No	8,092 4
5	0,000 0	8,219 6	No	8,828 6
6	0,000 0	8,977 6	No	9,527 8
7	0,000 0	9,735 7	No	10,196 1
8	0,015 2	10,493 7	No	10,837 4
9	0,773 2	11,251 8	No	11,454 8
10	1,531 2	12,009 8	No	12,050 3
11	2,289 3	12,767 8	No	12,625 7
12	3,047 3	13,525 9	No	13,182 2
13	3,805 4	14,283 9	No	13,720 6
14	4,563 4	15,042 0	No	14,241 7
15	5,321 4	15,800 0	No	14,745 9
16	6,079 5	16,558 0	No	15,233 7
17	6,837 5	17,316 1	No	15,705 1
18	7,595 6	18,030 0	No	16,160 2
19	8,353 6	18,030 0	No	16,599 0
20	9,111 6	18,030 0	No	17,021 1
21	9,869 7	18,030 0	No	17,426 3
22	10,627 7	18,030 0	No	17,814 1
$r_0 = 23$	11,385 8	18,030 0	No	18,183 9
24	18,030 0	N/A	Always	N/A
	Always reject at 24 failures or more.			

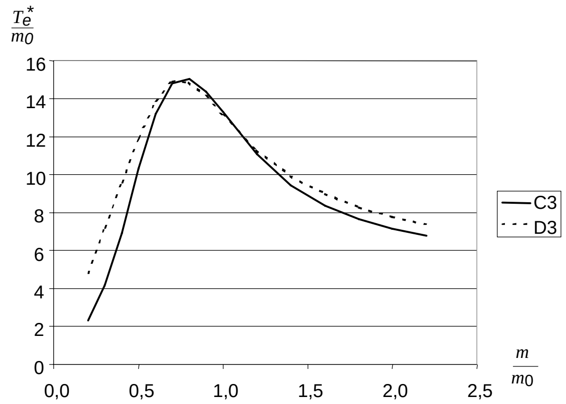

Figure D.10 – Expected test time to decision T_e^*

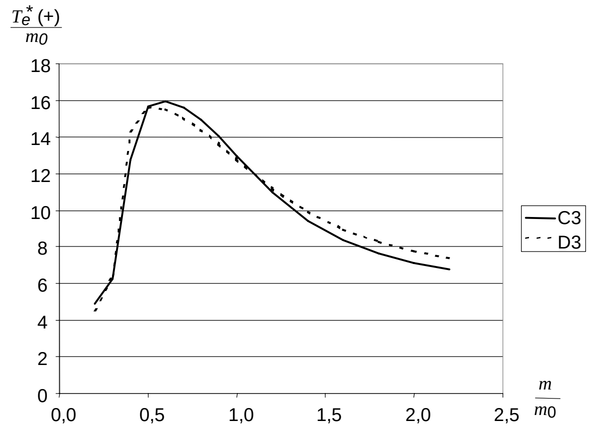

Figure D.11 – Expected test time to decision of acceptance $T_e^*(+)$

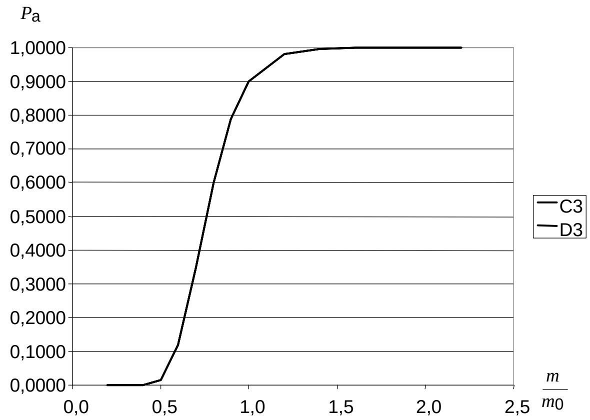

Figure D.12 – Operating characteristic P_a

Table D.8 – Expected test time to decision and operating characteristic P_a

	Plan C3		Plan D3				P_a	
m/m_0	T_e^*/m_0	$T_e^*(+)/m_0$	T_e^*/m_0	$T_e^*(+)/m_0$		m/m_0	Plan C3	Plan D3
0,2	2,279 1	4,889 3	4,800 0	4,551 2		0,2	0,000 0	0,000 0
0,3	4,152 5	6,242 3	7,200 0	6,422 1		0,3	0,000 0	0,000 0
0,4	6,920 1	12,759 5	9,598 8	14,294 0		0,4	0,000 4	0,000 3
0,5	10,295 5	15,650 0	11,948 4	15,586 3		0,5	0,015 8	0,015 6
0,6	13,183 2	15,972 7	13,908 6	15,505 1		0,6	0,119 3	0,119 4
0,7	14,790 4	15,625 2	14,912 4	15,058 8		0,7	0,346 6	0,346 9
0,8	15,042 6	14,932 3	14,848 0	14,404 8		0,8	0,600 7	0,600 9
0,9	14,369 1	14,011 9	14,104 9	13,618 1		0,9	0,789 8	0,789 9
1,0	13,279 6	12,974 4	13,116 6	12,772 8		1,0	0,900 0	0,900 0
1,2	11,071 8	10,967 5	11,274 0	11,170 4		1,2	0,980 7	0,980 7
1,4	9,446 8	9,422 2	9,926 5	9,902 9		1,4	0,996 5	0,996 5
1,6	8,365 2	8,360 0	8,979 9	8,975 0		1,6	0,999 4	0,999 4
1,8	7,638 1	7,637 0	8,296 6	8,295 6		1,8	0,999 9	0,999 9
2,0	7,129 0	7,128 8	7,785 3	7,785 1		2,0	1,000 0	1,000 0
2,2	6,756 8	6,756 7	7,390 2	7,390 2		2,2	1,000 0	1,000 0

D.5 Test plans C.4 and D.4 – α = 15 %, β = 5 %; D = 1,7

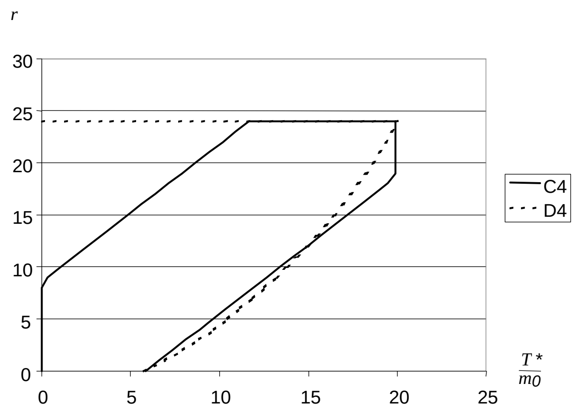

Figure D.13 – Accept and reject lines

Table D.9 – Accept and reject lines

Failures r	Accumulated test time T^*/m_0			
	Plan C4		Plan D4	
	Reject	Accept	Reject	Accept
0	0,000 0	5,836 6	No	5,732 0
1	0,000 0	6,594 6	No	6,946 2
2	0,000 0	7,352 7	No	7,960 5
3	0,000 0	8,110 7	No	8,865 7
4	0,000 0	8,868 8	No	9,697 2
5	0,000 0	9,626 8	No	10,473 0
6	0,000 0	10,384 8	No	11,204 0
7	0,000 0	11,142 9	No	11,897 4
8	0,000 0	11,900 9	No	12,558 0
9	0,303 2	12,659 0	No	13,189 4
10	1,061 3	13,417 0	No	13,794 1
11	1,819 3	14,175 0	No	14,374 2
12	2,577 3	14,933 1	No	14,931 2
13	3,335 4	15,691 1	No	15,466 1
14	4,093 4	16,449 2	No	15,979 8
15	4,851 5	17,207 2	No	16,472 9
16	5,609 5	17,965 2	No	16,945 8
17	6,367 5	18,723 3	No	17,398 7
18	7,125 6	19,481 3	No	17,831 5
19	7,883 6	19,900 0	No	18,244 3
20	8,641 7	19,900 0	No	18,636 7
21	9,399 7	19,900 0	No	19,008 2
22	10,157 7	19,900 0	No	19,358 1
23	10,915 8	19,900 0	No	19,685 7
$r_0 = 24$	11,673 8	19,900 0	No	19,989 8
25	19,900 0	N/A	Always	N/A
	Always reject at 25 failures or more.			

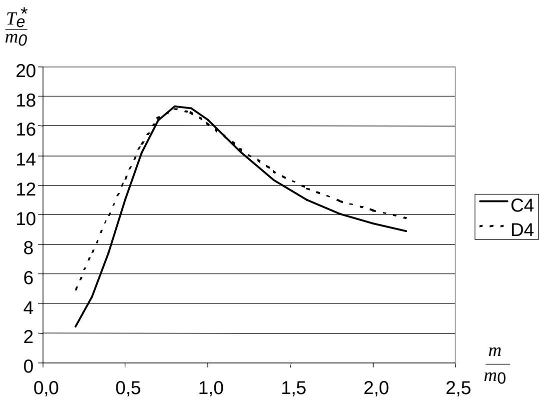

Figure D.14 – Expected test time to decision T_e^*

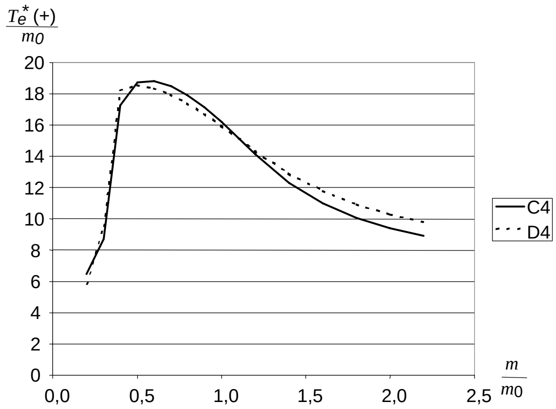

Figure D.15 – Expected test time to decision of acceptance $T_e^*(+)$

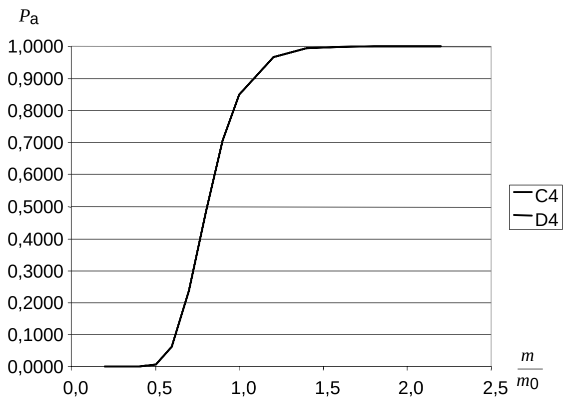

Figure D. 16 – Operating characteristic P_a

Table D.10 – Expected test time to decision and operating characteristic P_a

	Plan C4		Plan D4	
***m*/*m*$_0$**	T_e^*/m_0	$T_e^*(+)/m_0$	T_e^*/m_0	$T_e^*(+)/m_0$
0,2	2,447 5	6,442 7	5,000 0	5,830 8
0,3	4,460 1	8,723 0	7,500 0	9,557 4
0,4	7,420 7	17,275 0	9,999 9	18,229 0
0,5	10,990 5	18,772 1	12,488 0	18,570 3
0,6	14,178 5	18,817 0	14,806 3	18,360 2
0,7	16,365 1	18,499 1	16,483 7	17,960 5
0,8	17,318 9	17,929 2	17,152 6	17,407 7
0,9	17,199 5	17,136 4	16,938 8	16,723 1
1,0	16,408 3	16,174 8	16,224 5	15,945 4
1,2	14,225 3	14,096 5	14,451 9	14,328 7
1,4	12,338 3	12,302 0	12,945 7	12,912 8
1,6	10,990 5	10,982 2	11,812 0	11,804 5
1,8	10,055 0	10,053 2	10,960 9	10,959 3
2,0	9,390 9	9,390 5	10,307 8	10,307 4
2,2	8,902 4	8,902 3	9,794 2	9,794 1

	P_a	
***m*/*m*$_0$**	**Plan C4**	**Plan D4**
0,2	0,000 0	0,000 0
0,3	0,000 0	0,000 0
0,4	0,000 1	0,000 0
0,5	0,005 2	0,005 1
0,6	0,062 3	0,062 3
0,7	0,237 7	0,237 8
0,8	0,486 2	0,486 3
0,9	0,705 5	0,705 6
1,0	0,850 0	0,850 0
1,2	0,968 3	0,968 3
1,4	0,994 0	0,994 0
1,6	0,998 9	0,998 9
1,8	0,999 8	0,999 8
2,0	1,000 0	1,000 0
2,2	1,000 0	1,000 0

D.6 Test plans C.5 and D.5 – α = 20 %, β = 10 %; D = 1,7

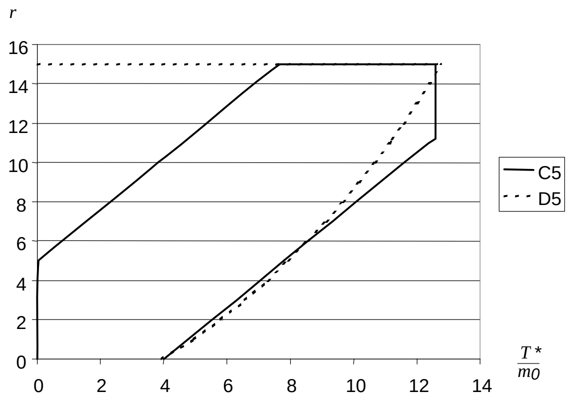

Figure D.17 – Accept and reject lines

Table D.11 – Accept and reject lines

Failures r	**Accumulated test time** T^*/m_0			
	Plan C5		**Plan D5**	
	Reject	**Accept**	**Reject**	**Accept**
0	0,000 0	4,023 2	No	3,949 5
1	0,000 0	4,781 2	No	4,965 6
2	0,000 0	5,539 3	No	5,818 1
3	0,000 0	6,297 3	No	6,580 4
4	0,000 0	7,055 4	No	7,280 4
5	0,0418	7,813 4	No	7,932 4
6	0,799 8	8,571 4	No	8,544 8
7	1,557 8	9,329 5	No	9,122 9
8	2,315 9	10,087 5	No	9,670 3
9	3,073 9	10,845 6	No	10,189 2
10	3,832 0	11,603 6	No	10,681 2
11	4,590 0	12,361 6	No	11,147 4
12	5,348 1	12,600 0	No	11,588 2
13	6,106 1	12,600 0	No	12,003 7
14	6,864 1	12,600 0	No	12,393 7
$r_0 = 15$	7,662 2	12,600 0	No	12,757 3
16	12,600 0	N/A	Always	N/A
	Always reject at 16 failures or more.			

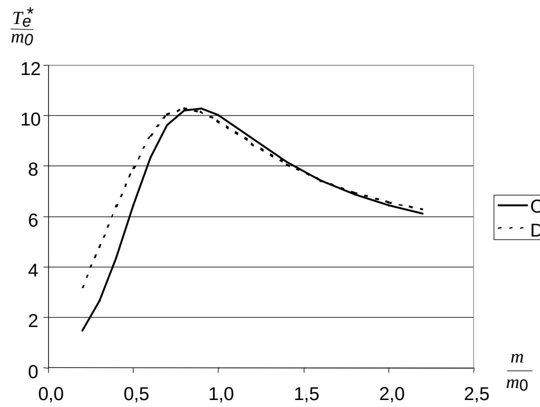

Figure D.18 – Expected test time to decision T_e^*

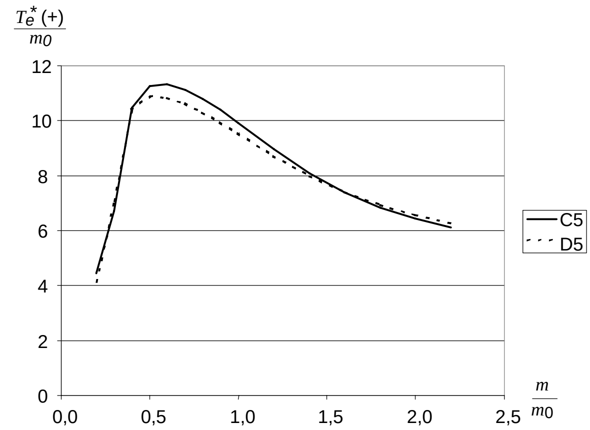

Figure D.19 – Expected test time to decision of acceptance $T_e^*(+)$

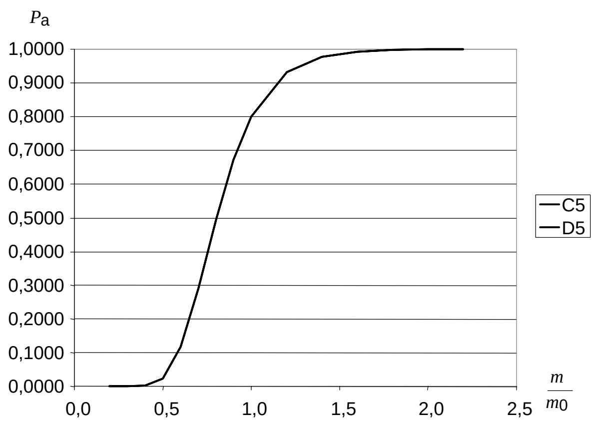

Figure D.20 – Operating characteristic P_a

Table D.12 – Expected test time to decision and operating characteristic P_a

	Plan C5		Plan D5	
m/m_0	T_e^*/m_0	$T_e^*(+)/m_0$	T_e^*/m_0	$T_e^*(+)/m_0$
0,2	1,453 4	4,442 0	3,200 0	4,132 1
0,3	2,643 8	6,722 0	4,800 0	7,086 9
0,4	4,357 8	10,456 2	6,397 3	10,387 9
0,5	6,423 9	11,260 5	7,943 0	10,886 2
0,6	8,313 0	11,309 9	9,238 2	10,842 9
0,7	9,607 6	11,118 8	10,041 1	10,622 6
0,8	10,215 9	10,796 8	10,303 6	10,308 1
0,9	10,279 6	10,388 2	10,162 4	9,935 9
1,0	10,005 9	9,926 3	9,797 1	9,532 7
1,2	9,075 5	8,963 3	8,884 5	8,720 7
1,4	8,151 0	8,092 8	8,070 0	7,997 6
1,6	7,413 5	7,389 1	7,435 6	7,407 0
1,8	6,857 1	6,847 7	6,951 6	6,940 7
2,0	6,439 1	6,435 5	6,577 9	6,573 7
2,2	6,120 4	6,119 0	6,283 4	6,281 7

	P_a	
m/m_0	Plan C5	Plan D5
0,2	0,000 0	0,000 0
0,3	0,000 0	0,000 0
0,4	0,001 2	0,001 1
0,5	0,022 4	0,022 5
0,6	0,115 9	0,115 9
0,7	0,292 8	0,292 6
0,8	0,497 1	0,496 8
0,9	0,673 1	0,672 9
1,0	0,800 0	0,800 0
1,2	0,932 1	0,932 3
1,4	0,977 9	0,978 1
1,6	0,992 7	0,992 9
1,8	0,997 5	0,997 6
2,0	0,999 1	0,999 2
2,2	0,999 7	0,999 7

D.7 Test plans C.6 and D.6 – α = 20 %, β = 20 %; D = 1,7

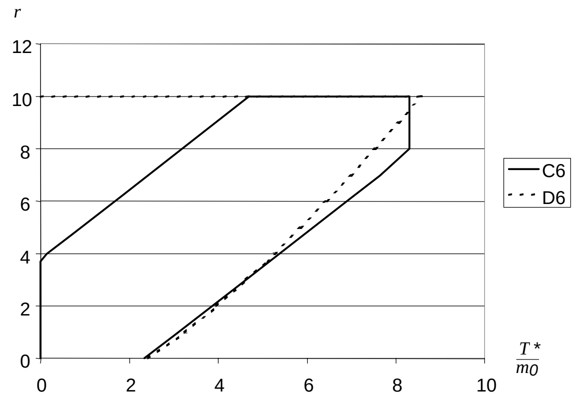

Figure D.21 – Accept and reject lines

Table D.13 – Accept and reject lines

Failures r	Accumulated test time $T*/m_0$			
	Plan C6		Plan D6	
	Reject	Accept	Reject	Accept
0	0,000 0	2,342 4	No	2,436 5
1	0,000 0	3,100 4	No	3,257 1
2	0,000 0	3,858 5	No	3,972 8
3	0,000 0	4,616 5	No	4,633 4
4	0,148 9	5,374 6	No	5,257 5
5	0,906 9	6,132 6	No	5,854 5
6	1,665 0	6,890 6	No	6,429 8
7	2,423 0	7,648 7	No	6,986 8
8	3,181 0	8,300 0	No	7,527 9
9	3,939 1	8,300 0	No	8,054 7
$r_0 = 10$	4,697 1	8,300 0	No	8,568 4
11	8,300 0	N/A	Always	N/A
	Always reject at 11 failures or more.			

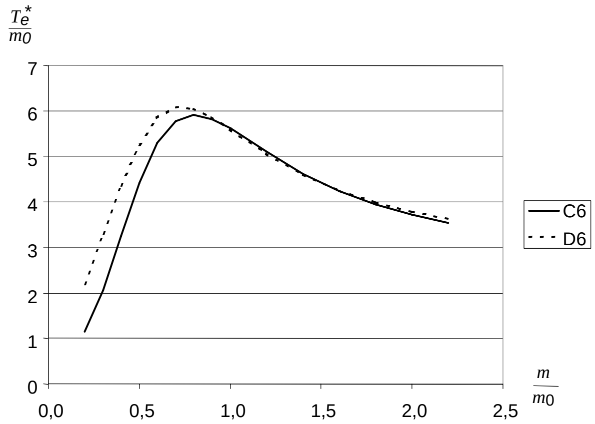

Figure D.22 – Expected test time to decision T_e^*

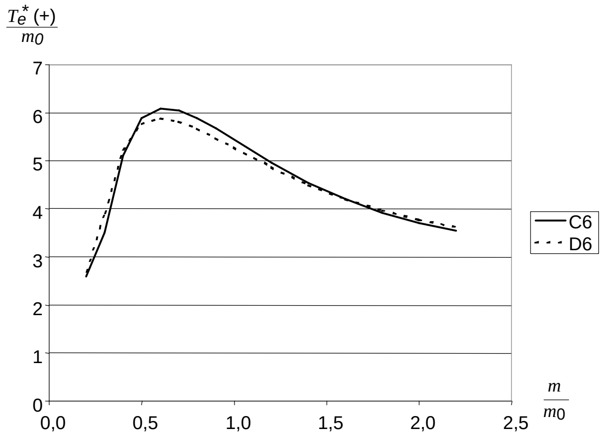

Figure D.23 – Expected test time to decision of acceptance $T_e^*(+)$

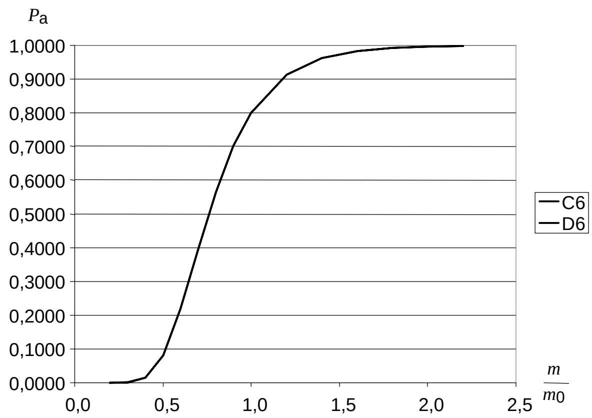

Figure D.24 – Operating characteristic P_a

Table D.14 – Expected test time to decision and operating characteristic P_a

	Plan C6		Plan D6				P_a	
m/m_0	T_e^*/m_0	$T_e^*(+)/m_0$	T_e^*/m_0	$T_e^*(+)/m_0$		m/m_0	Plan C6	Plan D6
0,2	1,145 9	2,587 4	2,200 0	2,700 2		0,2	0,000 0	0,000 0
0,3	2,064 2	3,513 7	3,297 9	3,916 6		0,3	0,001 0	0,000 8
0,4	3,248 2	5,114 3	4,360 6	5,241 1		0,4	0,014 9	0,014 5
0,5	4,427 0	5,888 3	5,261 0	5,768 0		0,5	0,080 8	0,080 7
0,6	5,302 9	6,091 5	5,845 5	5,885 2		0,6	0,219 3	0,219 3
0,7	5,775 5	6,049 7	6,080 7	5,823 6		0,7	0,396 1	0,395 8
0,8	5,909 2	5,893 7	6,048 1	5,676 2		0,8	0,564 9	0,564 5
0,9	5,821 5	5,681 6	5,858 1	5,486 7		0,9	0,700 9	0,700 6
1,0	5,615 3	5,445 1	5,598 0	5,279 9		1,0	0,800 0	0,800 0
1,2	5,097 9	4,969 3	5,057 8	4,869 3		1,2	0,913 6	0,914 0
1,4	4,621 0	4,547 5	4,605 7	4,507 1		1,4	0,962 6	0,963 2
1,6	4,240 2	4,201 9	4,258 6	4,209 2		1,6	0,983 4	0,983 8
1,8	3,947 4	3,927 9	3,995 5	3,970 9		1,8	0,992 3	0,992 7
2,0	3,722 4	3,712 6	3,793 7	3,781 3		2,0	0,996 3	0,996 5
2,2	3,547 6	3,542 5	3,635 8	3,629 4		2,2	0,998 1	0,998 3

D.8 Test plans C.7 and D.7 – α = 30 %, β = 20 %; D = 1,7

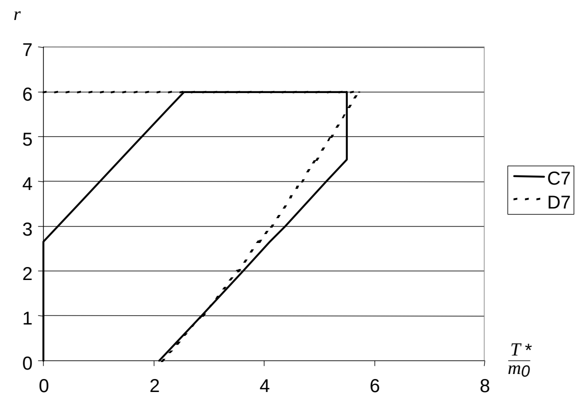

Figure D.25 – Accept and reject lines

Table D.15 – Accept and reject lines

Failures r	Accumulated test time T^*/m_0			
	Plan C7		Plan D7	
	Reject	Accept	Reject	Accept
0	0,000 0	2,102 1	No	2,144 1
1	0,000 0	2,860 2	No	2,887 7
2	0,000 0	3,618 2	No	3,533 4
3	0,266 5	4,376 2	No	4,126 1
4	1,024 5	5,134 3	No	4,682 3
5	1,782 5	5,500 0	No	5,210 3
$r_0 = 6$	2,540 6	5,500 0	No	5,714 6
7	5,500 0	N/A	Always	N/A
	Always reject at 7 failures or more.			

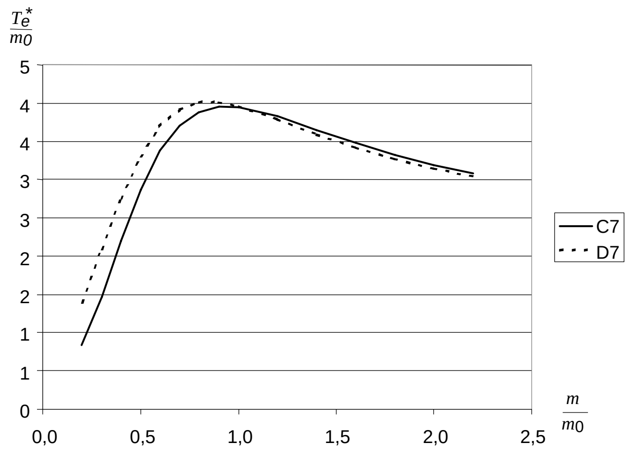

Figure D.26 – Expected test time to decision T_e^*

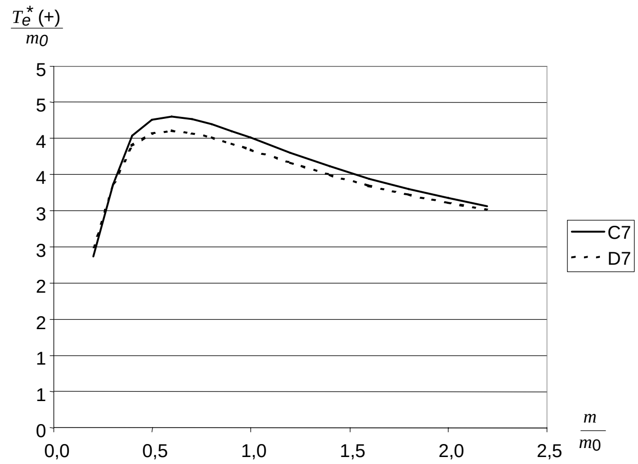

Figure D.27 – Expected test time to decision of acceptance $T_e^*(+)$

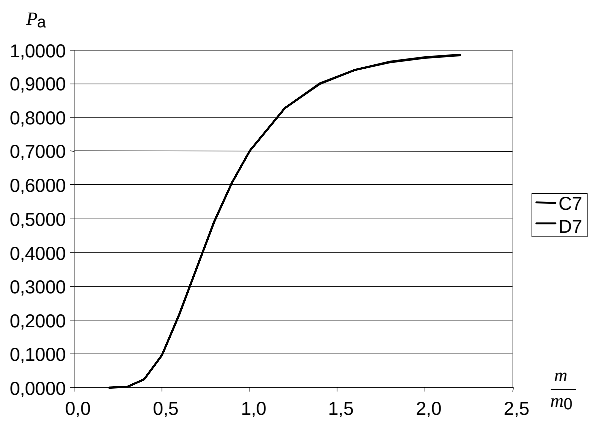

Figure D.28 – Operating characteristic P_a

Table D.16 – Expected test time to decision and operating characteristic P_a

	Plan C7		Plan D7	
m/m_0	T_e^*/m_0	$T_e^*(+)/m_0$	T_e^*/m_0	$T_e^*(+)/m_0$
0,2	0,832 1	2,370 5	1,400 0	2,497 3
0,3	1,467 0	3,358 6	2,096 4	3,377 3
0,4	2,194 3	4,033 2	2,757 2	3,895 9
0,5	2,864 7	4,257 6	3,309 5	4,076 7
0,6	3,375 9	4,300 1	3,697 5	4,111 2
0,7	3,706 6	4,265 7	3,920 4	4,079 7
0,8	3,885 0	4,194 6	4,012 4	4,015 6
0,9	3,952 8	4,105 1	4,015 3	3,935 1
1,0	3,948 1	4,006 9	3,964 0	3,847 3
1,2	3,826 0	3,805 8	3,789 1	3,668 4
1,4	3,652 3	3,616 3	3,595 6	3,500 9
1,6	3,479 9	3,447 5	3,420 4	3,352 6
1,8	3,326 2	3,301 3	3,271 5	3,224 7
2,0	3,194 4	3,176 5	3,147 6	3,115 6
2,2	3,082 8	3,070 3	3,04 8	3,022 9

	P_a	
m/m_0	Plan C7	Plan D7
0,2	0,000 0	0,000 0
0,3	0,002 3	0,002 4
0,4	0,024 8	0,025 2
0,5	0,096 3	0,096 7
0,6	0,215 7	0,215 6
0,7	0,355 7	0,355 2
0,8	0,490 6	0,489 9
0,9	0,606 5	0,606 1
1,0	0,700 0	0,700 0
1,2	0,827 7	0,828 4
1,4	0,900 2	0,901 3
1,6	0,940 8	0,942 1
1,8	0,964 0	0,965 2
2,0	0,977 4	0,978 5
2,2	0,985 5	0,986 4

D.9 Test plans C.8 and D.8 – α = 30 %, β = 30 %; D = 1,7

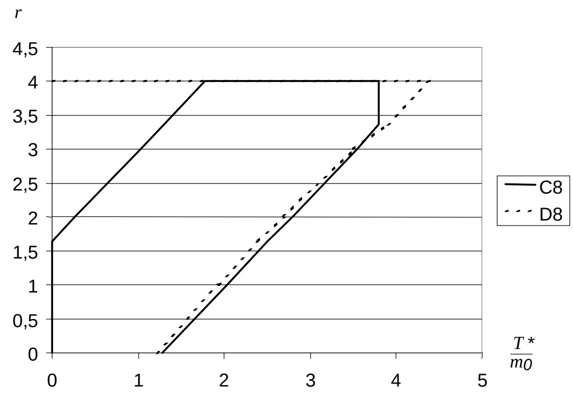

Figure D.29 – Accept and reject lines

Table D.17 – Accept and reject lines

Failures r	**Accumulated test time** T^*/m_0			
	Plan C8		**Plan D8**	
	Reject	**Accept**	**Reject**	**Accept**
0	0,000 0	1,275 7	No	1,219 4
1	0,000 0	2,033 8	No	1,930 7
2	0,263 6	2,791 8	No	2,689 1
3	1,021 6	3,549 9	No	3,508 7
$r_0 = 4$	1,779 6	3,800 0	No	4,388 1
5	3,800 0	N/A	Always	N/A
	Always reject at 5 failures or more.			

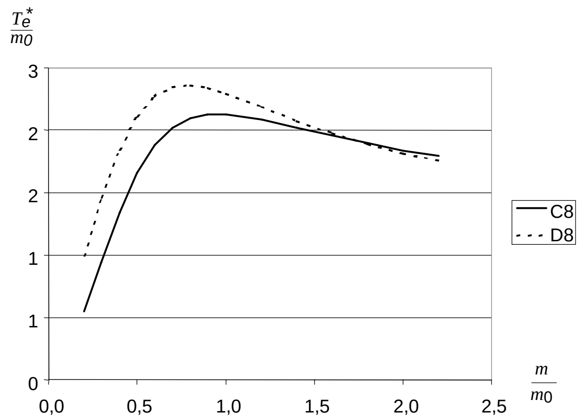

Figure D.30 – Expected test time to decision T_e^*

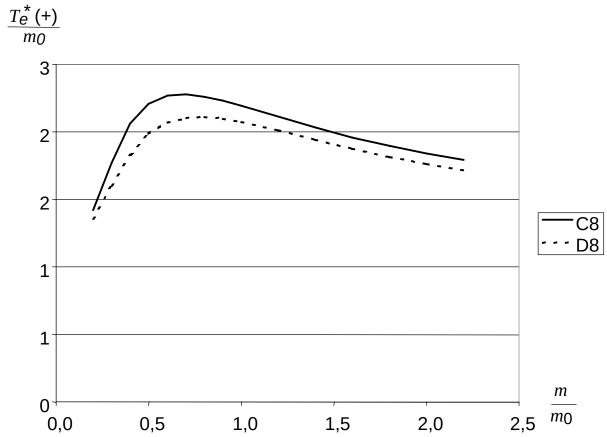

Figure D.31 – Expected test time to decision of acceptance $T_e^*(+)$

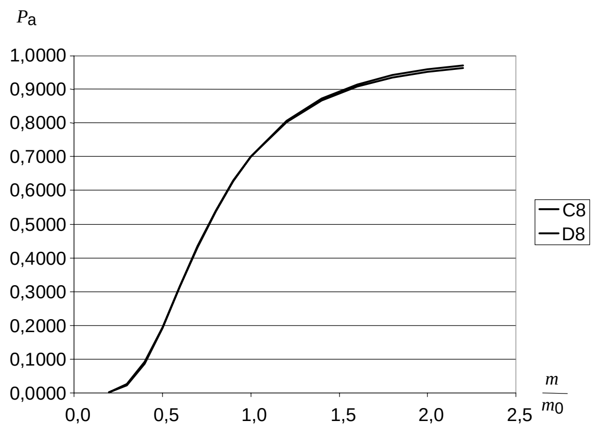

Figure D.32 – Operating characteristic P_a

Table D.18 – Expected test time to decision and operating characteristic P_a

	Plan C8		Plan D8				P_a	
m/m_0	T_e^*/m_0	$T_e^*(+)/m_0$	T_e^*/m_0	$T_e^*(+)/m_0$		m/m_0	Plan C8	Plan D8
0,2	0,548 2	1,419 2	0,997 4	1,360 7		0,2	0,002 0	0,002 7
0,3	0,943 6	1,766 6	1,463 9	1,603 3		0,3	0,022 8	0,026 9
0,4	1,339 2	2,059 9	1,845 9	1,831 0		0,4	0,086 5	0,092 3
0,5	1,660 2	2,209 9	2,112 9	1,984 3		0,5	0,191 5	0,194 7
0,6	1,883 5	2,268 2	2,270 8	2,068 7		0,6	0,314 7	0,314 3
0,7	2,021 0	2,277 4	2,344 2	2,105 3		0,7	0,434 7	0,432 2
0,8	2,094 6	2,260 6	2,359 8	2,111 5		0,8	0,540 5	0,537 7
0,9	2,124 7	2,230 1	2,339 3	2,099 1		0,9	0,628 7	0,627 0
1,0	2,126 9	2,192 7	2,298 2	2,075 5		1,0	0,700 0	0,700 0
1,2	2,086 8	2,111 3	2,190 3	2,012 4		1,2	0,802 0	0,805 4
1,4	2,023 6	2,031 9	2,079 0	1,943 3		1,4	0,866 0	0,871 9
1,6	1,957 2	1,959 9	1,978 7	1,877 0		1,6	0,906 8	0,913 9
1,8	1,895 3	1,896 2	1,892 5	1,816 6		1,8	0,933 3	0,940 8
2,0	1,839 9	1,840 7	1,819 8	1,762 9		2,0	0,951 0	0,958 4
2,2	1,791 4	1,792 4	1,758 6	1,715 7		2,2	0,963 1	0,970 2

Annex E
(informative)

Examples and mathematical references for sequential test plans

E.1 Symbols

In addition to the symbols given in 3.2.2 the following symbols are used:

A	constant
B	constant
a	the accept line's intersection with the r axis
b	the accept and reject line's slope
b^*	$b^* = b \times m_0$, slope
c	the reject line's intersection with the r axis
E_r	expected number of failures
E_t	expected average time to arrive to a decision for n items under test
h	iteration variable for design of OC-curves
h_0	parameter used for computing E_r
h_1	parameter used for computing E_r
$P(r)$	probability ratio as a function of number of failures
$P_0(r)$	probability of having exactly r failures in the time T^* for true $m = m_0$
$P_1(r)$	probability of having exactly r failures in the time T^* for true $m = m_1$
$P_r(r)$	probability ratio
s	parameter used for computing E_r
t^*	test time for each test item
t_a^*	test time for each test item, stated as acceptance criterion
t_r^*	test time for each test item, stated as reject criterion
k	summation variable for failures
$\chi^2_{\alpha;\,2r_0}$	α fractile of the χ^2 distribution with $2r_0$ degrees of freedom
$\chi^2_{1-\beta;\,2r_0}$	$1-\beta$ fractile of the χ^2 distribution with $2r_0$ degrees of freedom

E.2 Example of a sequential test

Specified: n = 500 components that are placed on test with replacement. Producer and consumer agree on $\lambda_0 = 9 \times 10^{-7}$ h^{-1} and $\alpha = \beta$ = 10 % and D = 3.

To be derived: Sequential test plan and simulated examples.

Procedures: $m_0 = \dfrac{1}{\lambda} = 1{,}11 \times 10^6$ h (= 126,8 years).

Since replacement is used the accumulated test time for 500 components, T^*, is:

$$T^* = nt^* = 500\, t^*$$

where t^* is the test time per item.

Test plan A.3 satisfies the specified α, β and D while T_a^* and T_r^* are the accumulated test times for accept and reject, respectively, they are stated as multiples of m_0 that is $\dfrac{T_a^*}{m_0}$ and $\dfrac{T_r^*}{m_0}$

Each time a failure occurs, the point $(\dfrac{T^*}{m_0}, r) = (\dfrac{nt^*}{m_0}, r)$ should be plotted in the diagram and it should be noted whether it falls:

– on or above the reject border lines, *reject*;
– between the border lines, *continue testing*;
– on or below the accept border lines, *accept.*

The border lines have the following equations (see Table E.2):

Reject lines: $r = 1{,}63 + 1{,}82\left(\dfrac{T^*}{m_0}\right)$, $0 \le \dfrac{T^*}{m_0} \le 2{,}4$ and r = 6, $2{,}4 \le \dfrac{T^*}{m_0} < 3{,}15$

Accept lines: $r = -2{,}0 + 1{,}82\left(\dfrac{T^*}{m_0}\right)$, $1{,}10 \le \dfrac{T^*}{m_0} \le 3{,}74$ and $3{,}74 < r \le 6$, $\dfrac{T^*}{m_0} = 3{,}15$

The abscissa $\dfrac{T^*}{m_0}$ can be changed to t^* by multiplying by $\dfrac{m_0}{n}$ = $1{,}11 \times 10^6$ / 500 = 2 222 h.

Instead of plotting in the diagram, Table A.3 in Annex A can be used possibly by adding two columns showing t_a^* and t_r^* in test hours, that is $t_a^* = \left[\dfrac{T_a^*}{m_0}\right] \times \dfrac{m_0}{n}$ and $t_r^* = \left[\dfrac{T_r^*}{m_0}\right] \times \dfrac{m_0}{n}$ where the quantities in the square brackets are the stated values.

An easy way to perform a sequential test is to put the table into a spreadsheet program with graphics. An example with fictive data is shown in Table E.1 and Figure E.1.

Table E.1 – Example for a sequential test using test plan A.3 (with example data)

$\alpha = \beta$ = 10 %, D = 3, m_0 = 1,11×10^6 h, $t^* = 2\,222 \times \dfrac{T^*}{m_0}$ h							
Test plan A.3					**Test result**		
r	$\frac{T_r^*}{m_0}$ **Reject** (equal or less)	$\frac{T_a^*}{m_0}$ **Accept** (equal or more)	t_r^* in hours **Reject** (equal or less)	t_a^* in hours **Accept** (equal or more)	$\frac{T^*}{m_0}$	T^* in hours	**Decision**
0	0	1,098	0	2 440	0,000	0	Continue
1	0	1,648	0	3 658	0,739	1 641	Continue
2	0,203	2,197	451	4 878	1,071	2 380	Continue
3	0,752	2,747	1 678	6 098	1,211	2 688	Continue
4	1,301	3,152	2 890	6 998	1,481	3 288	Continue
5	1,851	3,152	4 110	6 998	1,598	3 548	Reject
$r_0 = 6$	2,400	3,152	5 328	6 998			
7	3,152	N/A	6 998	N/A			

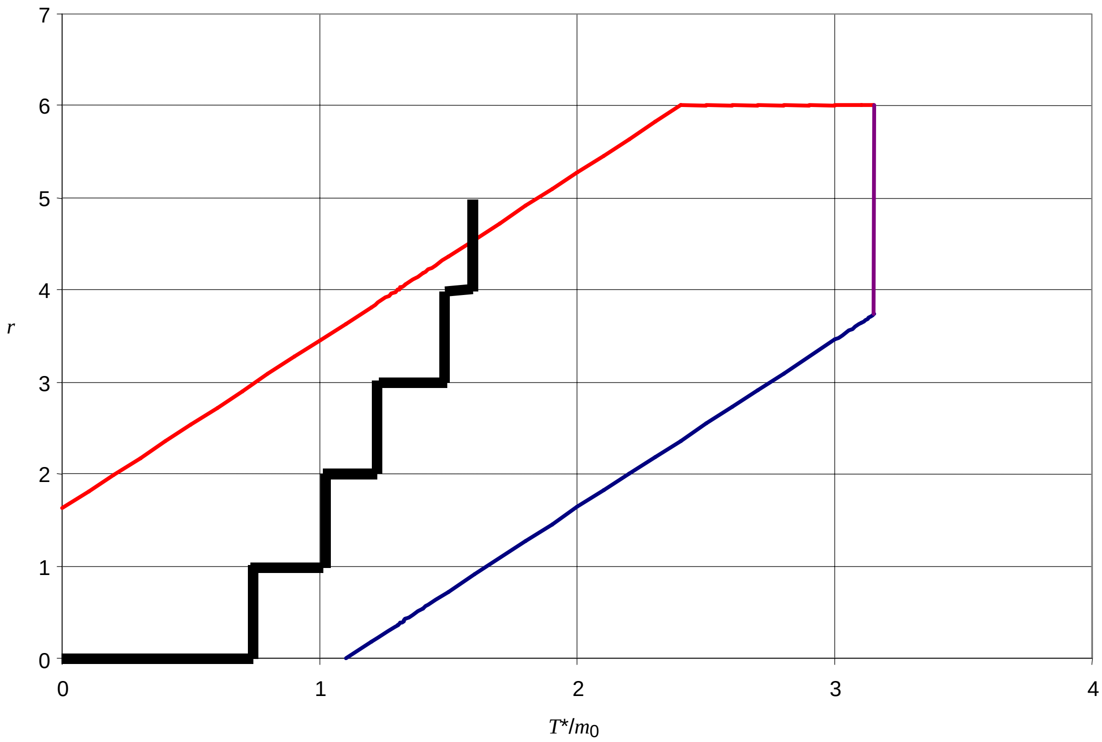

Figure E.1 – Example of a sequential test using test plan A.3 – $\alpha = \beta$ = 10 %, *D* = 3, m_0 = 1,11×10⁶ h; *r* versus t^*

E.3 Mathematical references for sequential test plans

E.3.1 Determination of the test plan

For a test item with an unknown *m*, probability of failing *r* times in an accumulated time of operation *t* is:

$$P(r) = \left(\frac{t}{m}\right)^r \left(\frac{e^{-t/m}}{r!}\right) \tag{E.1}$$

The test has to prove that $m \geq m_0$ against $m < m_0$. This can be done by Wald's Sequential Probability Ratio Test (SPRT) [3] by testing the hypothesis $H_0 : m = m_0$ against the hypothesis $H_1 : m = m_1$.

It is required that

– the probability of accepting $m \leq m_1$ is less than or equal to the consumer's risk β, and

– that probability of rejecting $m \geq m_0$ is less than or equal to the producer's risk α.

During this test, the probability ratio of the two probabilities is (see also Equation (E.5))

$$P_r(r) = P_1(r)/P_0(r) \tag{E.2}$$

The probability of having *r* failures in the accumulated test time T^* if the true *m* is equal to m_1 is:

$$P_1(r) = \left(\frac{T^*}{m_1}\right)^r \frac{\exp\left(-T^*/m_1\right)}{r!} \quad \text{(E.3)}$$

The probability of having *r* failures in the accumulated test time T^* if the true *m* is equal to m_0 is:

$$P_0(r) = \left(\frac{T^*}{m_0}\right)^r \frac{\exp\left(-T^*/m_0\right)}{r!} \quad \text{(E.4)}$$

The probability ratio between the two probabilities is then:

$$P_r(r) = \frac{P_1(r)}{P_0(r)} = D^r \cdot \exp\left[-\left(\frac{1}{m_1} - \frac{1}{m_0}\right) \cdot T^*\right] \quad \text{(E.5)}$$

This value is evaluated as being between the two constant values:

$$\text{B} \leq P_r(r) \leq \text{A} \quad \text{(E.6)}$$

Probability ratio in test is computed continuously and is compared to the two predetermined constants, A and B, using the following decision rules:

- if $P_r(r) \leq$ B, accept and stop testing;
- if $P_r(r) \geq$ A, reject and stop testing;
- if B < $P_r(r)$< A, continue the test.

The constants A and B are

$$\text{A} = \frac{1-\beta}{\alpha} \cdot \frac{D+1}{2 \cdot D}, \quad \text{(E.7)}$$

$$\text{B} = \frac{\beta}{1-\alpha} \quad \text{(E.8)}$$

The constant A contains the correction factor $(D+1)/(2 \cdot D)$. This factor is used to better fulfil the nominal risks (see [4]). In the equation above, *D* is known as a discrimination ratio:

$$D = \frac{m_0}{m_1} = \frac{\lambda_1}{\lambda_0} \quad \text{(E.9)}$$

With Equations (E.7) and (E.8) follows:

$$\text{B} < D^r \cdot \exp\left[-\left(\frac{1}{m_1} - \frac{1}{m_0}\right) \cdot T^*\right] < \text{A} \quad \text{(E.10)}$$

Taking logs (to the base e), it follows that:

$$\ln(\mathrm{B}) < r \cdot \ln(D) - \left(\frac{1}{m_1} - \frac{1}{m_0}\right) \cdot T^* < \ln(\mathrm{A}) \tag{E.11}$$

Rearranging the above equation by adding $\left(\frac{1}{m_1} - \frac{1}{m_0}\right) \cdot T^*$ to each term and by dividing all terms by $\ln(D)$ it follows that:

$$\frac{\ln(\mathrm{B})}{\ln(D)} + \frac{\left(\frac{1}{m_1} - \frac{1}{m_0}\right)}{\ln(D)} \cdot T^* < r < \frac{\ln(\mathrm{A})}{\ln(D)} + \frac{\left(\frac{1}{m_1} - \frac{1}{m_0}\right)}{\ln(D)} \cdot T^* \tag{E.12}$$

The above equation (E.13) is re-written as:

$$a + b \cdot T^* < r < c + b \cdot T^* \tag{E.13}$$

where $r = a + b \cdot T^*$ is the accept line and $r = c + b \cdot T^*$ is the reject line.

The constants are:

$$a = \frac{\ln(\mathrm{B})}{\ln(D)} \tag{E.14}$$

$$c = \frac{\ln(\mathrm{A})}{\ln(D)} \tag{E.15}$$

$$b = \frac{\left(\frac{1}{m_1} - \frac{1}{m_0}\right)}{\ln(D)} = \frac{D - 1}{m_0 \cdot \ln(D)} \tag{E.16}$$

The test truncation time is calculated using the following:

$$T_t^* = \frac{m_0 \chi^2_{\alpha;\, 2r_0}}{2} \tag{E.17}$$

where the test truncation failure number r_0 is determined from the ratio:

$$\frac{\chi^2_{\alpha;\, 2r_0}}{\chi^2_{1-\beta;\, 2r_0}} \geq \frac{m_1}{m_0} = \frac{1}{D} \tag{E.18}$$

The chi-square variables having $2r_0$ degrees of freedom can be found in statistical tables or spreadsheets that have statistical functions. The value of r_0 is determined to obtain the two chi-square functions having a ratio greater or equal to $1/D$.

The minimum test time for acceptance without a test failure is:

$$T^*_{\mathrm{a,\,min}} = -\frac{a}{b} \tag{E.19}$$

E.3.2 Determination of operating characteristic curve

The operating characteristics curve is determined by the following approximation [3].

$$P_{\mathrm{a}}(h) = \frac{\mathrm{A}^h - 1}{\mathrm{A}^h - \mathrm{B}^{\mathrm{h}}} \tag{E.20}$$

The value *h* is found from the following equation:

$$m = m_0 \cdot \frac{D^{\mathrm{h}} - 1}{h \cdot (D-1)} \tag{E.21}$$

E.3.3 Determination of expected test time

a) Tests with replacement of failed items

The expected test time for $n \geq 1$ items under test with replacement is derived as follows:
Expected number of failures in test to reach a decision is:

$$E_r = \frac{h_1 - P_{\mathrm{a}}(h_0 - h_1)}{s - m} \tag{E.22}$$

where

$$h_1 = m_0 \cdot \frac{\ln(\mathrm{A})}{D-1} \tag{E.23}$$

$$h_0 = m_0 \cdot \frac{\ln(\mathrm{B})}{D-1} \tag{E.24}$$

$$s = m_0 \cdot \frac{\ln(D)}{D-1} = \frac{1}{b} \tag{E.25}$$

When the expressions of Equations (E.20) through (E.22) are substituted into the Equation (E.22) the expected number of failures is then:

$$E_r = m_0 \cdot \frac{P_{\mathrm{a}}(\ln(\mathrm{A}) - \ln(\mathrm{B})) - \ln(\mathrm{A})}{m \cdot (D-1) - m_0 \cdot \ln(D)} \tag{E.26}$$

The average expected test time to reach a decision is:

$$E_t = \frac{m}{n} \cdot E_r \tag{E.27}$$

The expected test time in terms of m_0 for tests with replacement is with $T_\mathrm{e}^* = n \cdot E_\mathrm{t} = m \cdot E_\mathrm{r}$:

$$\frac{T_\mathrm{e}^*}{m_0} = \frac{m \cdot E_r}{m_0} = m \cdot \frac{P_\mathrm{a}\left[\ln(\mathrm{A}) - \ln(\mathrm{B})\right] - \ln(\mathrm{A})}{m \cdot (D-1) - m_0 \cdot \ln(D)} \tag{E.28}$$

b) Tests without replacement of failed items

When the test is without replacement, the average test time to arrive at a decision for $n \geq r_0$ items under test is:

$$E_t = m \cdot \ln\left(\frac{n}{n - E_r}\right) \tag{E.29}$$

with E_r from Equation (E.26).

E.4 Border lines and coordinates

The following table shows the extreme coordinates of the border lines and the constants for the border line equations for acceptance, $r_a = a + b^* \cdot \frac{T^*}{m_0}$ and for rejection, $r_r = c + b^* \cdot \frac{T^*}{m_0}$ with $b^* = b \cdot m_0 = \frac{D-1}{\ln(D)}$.

Table E.2 – Constants for border line equations and their coordinates for sequential test plans A. 1 to A. 9

Test plan	$\alpha = \beta$ %	D	Constants for border line equations			Coordinates for border lines $\left\{T^*/m_0 ;\ r\right\}$			
			a	b^*	c	Reject line		Accept line	
A.1	10	1,5	–5,42	1,2334	4,97	0; 4,97	29,22 ; 41	4,39 ; 0	33,09 ; 35,32
A.2	10	2	–3,17	1,4427	2,755	0; 2,76	8,49; 15	2,20; 0	10,30; 11,69
A.3	10	3	–2,00	1,8205	1,631	0; 1,63	2,40; 6	1,10; 0	3,15; 3,74
A.4	10	5	–1,37	2,4853	1,05	0; 1,05	0,79; 3	0,55; 0	1,10; 1,37
A.5	20	1,5	–3,42	1,2332	2,97	0; 2,97	12,19; 18	2,77; 0	14,37; 14,30
A.6	20	2	–2,00	1,4427	1,58	0; 1,58	3,75; 7	1,39; 0	4,73; 4,83
A.7	20	3	–1,26	1,8205	0,89	0; 0,89	1,16; 3	0,69; 0	1,54; 1,53
A.8	30	1,5	–2,09	1,2332	1,64	0; 1,64	4,35; 7	1,69; 0	5,41; 4,58
A.9	30	2	–1,22	1,4427	0,81	0; 0,81	1,52; 3	0,85; 0	1,91; 1,54

Annex F
(informative)

Design of sequential test plans using a spreadsheet program

F.1 List of symbols

In addition to the symbols given in 3.2.2 and in Clause E.1, the following symbols are used in this annex :

a the acceptance line's intersection with the r axis

b the slope of the accept and the reject lines

c the reject line's intersection with the r axis

n number of items on test

E_r expected number of failures

E_t expected average time to arrive to a decision for n items under test

F.2 Construction of sequential plans using software tools

F.2.1 Construction of sequential test plans

The test plans with the decision lines as well as the truncation time can be constructed and optimized using commercial software or any commercially available spreadsheet that allows insertion of functions and equations.

Given the worldwide use of computers and the development of state-of-the-art standard computer software, the spreadsheet is developed to ease the process of curve development and use, especially in cases where unique parameters are given, those that cannot be readily found in a table or prepared graph.

The spreadsheet is constructed with the advantage of embedded equations that use parameters given in the spreadsheet such as: producers and consumer's risks (α and β), discrimination ratio (D), and m_0. By changing the parameter values, and by determination of the test termination time T_t^* the necessary plot is re-drawn from the initial plot.

The sequential test example provided in Clause F.3 provides step-by-step construction of a spreadsheet for a sequential probability ratio test, SPRT.

This methodology may make the use of pre-prepared graphs and tables obsolete unless computer access is not available. Test plans described in the following clauses of this standard can also be graphed using the embedded graphing methodology for the determined test parameters (test plan number), or they can be used for any value of the test parameters, regardless of the existence of the pre-prepared charts.

Once set up with all embedded equations, the graphs can be re-drawn for any required parameters.

When calculated and plotted with the spreadsheet, test plans A.1 to A.9 may slightly differ from those shown in the Annex A. The difference is a result of rounding of the chi-square functions, which in turn might result into selection of different numbers of the maximum allowable number of test failures. The spreadsheet calculations disregard rounding of the numbers; therefore, the resultant calculations and graphs are more precise.

When preparing the test plans in a spreadsheet, as explained in Clause F.3, the last number of failures for rejection might appear slightly greater or less than the closes integer. This is because of the test gradation on the x axis. The gradation or granularity occurs because of the interval assigned to the increase of the test time. The calculated values for the number of failures at that corresponding time may end up being slightly above or below the integer. The solution would be to adjust the increase of the test time in the proximity of the rejection number of failures so that the plotting can stop at the integer, or to replace the end (reject) values by the corresponding integer number of failures.

F.2.2 Construction of the OC curves and the estimated test time for the sequential tests using a spreadsheet

The OC curves are based on equations in Clause F.4. As OC curves represent a plot of probability of acceptance as a function of m, the spreadsheet includes assigning values to the variable h such that the resultant m values would be within the range (and exceeding that range) of the values of m_1 and m_0. Values of the variable m are then calculated from the values of its variable, h, and the probability of acceptance is calculated from the values of the variable h and then plotted against values of m.

The procedure is explained in Clause F.4.

Determination of the estimated test time is also explained in Annex E and use of a spreadsheet to prepare graphs as a function of m is shown in Clauses F.3 and F.4.

F.3 Construction of sequential test plans using software tools

To set up a spreadsheet, enter headings for specific parameters and the appropriate equations as shown in Figures F.3 and F.4. The example in Figure F.4 is the spreadsheet used for the preparation of test plan A.5 shown in Annex A.

In Figure F.1, parameter values are set in cells C3, D3, F3 and G3, while the equation shown in E2 is embedded into the E2 to calculate discrimination ratio from m_0 and m_1.

For different values of parameters, all other values in the spreadsheet, once all equations are embedded, will be recalculated.

	A	B	C	D	E	F	G	H	I
1	r_0	$2r$	β	α	D	m_0	m_1	T_t*	T_t*/m_1
2					**=$F$3/$G$3**				
3			**0,2**	**0,2**	**1,5**	**3 000**	**2 000**		
4			**=$C$3**	**=1–$D$3**	**=1/$E$3**				
5			**0,2**	**0,8**	**0,667**				

Figure F.1 – Beginning of the spreadsheet prepared to obtain a sequential test graph

NOTE Symbols $ shown in the embedded equations of the spreadsheet denote that the value is fixed and that only the value of that specific cell address is used. This symbol may be different for the different spreadsheet programs that might be used.

Equations shown in C4 to E4 are those that need to be written into C5 to E5, typed in the same way only without the quotation marks. Such marks are inserted so as to display the equations in the spreadsheet.

Further into the spreadsheet, the remaining equations needed for the upper and lower line of the sequential test (SPRT) graph are inserted as shown in Figure F.2.

	J	K	L	M	N
1	A	B	*a*	*b*	*c*
2	$A = \frac{(1-\beta)(D+1)}{2\alpha D}$	$B = \frac{\beta}{(1-\alpha)}$	$a = \frac{\ln(B)}{\ln\left(\frac{m_0}{m_1}\right)}$	$b = \frac{\left(\frac{1}{m_1} - \frac{1}{m_0}\right)}{\ln\left(\frac{m_0}{m_1}\right)}$	$c = \frac{\ln(A)}{\ln\left(\frac{m_0}{m_1}\right)}$
3	**=((1-$C$3)*($E$3+1))/(2*($D$3)*($E$3))**	**=$C$3/(1–$D$3)**	**=LN($K$4)/LN($E$3)**	**=((1/$G$3)–(1/$F$3))/LN($E$3)**	**=LN($J$4)/LN($E$3)**
4	**3,33**	**0,25**	**-3,419 023**	**0,000 411**	**2,969 362**

Figure F.2 – Continuation of parameters calculation for the lines necessary for the SPRT graph

Values in J4 to N4 represent equations given in J3 to N3 respectively.

When parameters *a*, *b*, and *c* are determined, then the lines can be plotted as a function of test time. The data for the accept and reject lines are shown in Figure F.3.

	O	P	Q	R
1			*a*+*bt*	*c*+*bt*
2		=O5/$F$3	=$L$4+$M$4*O5	=$N$4+$M$4*O5
3	Variable	Variable		
4	***t*** **(hours)**	$\boldsymbol{t/m_0}$ **(time = Number of** $\boldsymbol{m_0}$**)**	***a*+*bt***	***c*+*bt***
5	0	0	-3,419 023	2,969 362
6	200	0,1	-3,336 812	3,051 572

Figure F.3 – Calculations of accept and reject line for the SPRT graph

In Figure F.3 values on Q5, Q6, etc. and in R5, R6, etc, are equations inserted into the spreadsheet and those are shown in Q4 and R4 respectively.

To calculate termination time T_t^* (Equation E.16), it is necessary to determine the two chi-square functions shown in Equation (E.17). Those are also embedded into the spreadsheet as shown in Figure F.4, Plotted are the chi-square functions necessary for Equation (15) and their ratio, as a function of number of failures *r*, or double number of failures (2*r*).

	A	B	C	D	E	F	G	H
1	r_0	$2r$	α	β	D	m_0	m_1	T_t*
2					=$F$3/$G$3			
3			0,2	0,2	1,5	3 000	2 000	
4					"=1/$E$3"			
5					0,666667			
6								
7								
8								
9	=B10/2=r/2	$2r$	=CHIINV($C$3,$B10)	=CHIINV($D$5,$B10)	=D10/C10			=($F$3*D10)/2
10	0,5	1	1,642 376	0,064185	0,039			96,28
11	1	2	3,218 879	0,446287	0,139			669,43
12	1,5	3	4,641 630	1,005173	0,217			1 507,76
13	2	4	5,988 616	1,648776	0,275			2 473,16
14	2,5	5	7,289 273	2,342532	0,321			3 513,80
15	3	6	8,558 058	3,070088	0,359			4 605,13
16	3,5	7	9,803 248	3,822320	0,399			5 733,48
17	4	8	11,030 090	4,593572	0,417			6 890,36
18	4,5	9	12,242 141	5,380055	0,439			8 070,08
19	5	10	13,441 963	6,179076	0,460			9 268,61
20	5,5	11	14,631 420	6,988672	0,478			10 483,01
21	6	12	15,811 990	7,807329	0,494			11 710,99
22	6,5	13	16,984 793	8,633 863	0,508			12 950,80
23	7	14	18,150 767	9,467 329	0,522			14 200,99
24	7,5	15	19,310 653	10,306 960	0,534			15 460,44
25	8	16	20,465 074	11,152 120	0,545			16 728,18
26	8,5	17	21,614 562	12,002 260	0,555			18 003,40
27	9	18	22,759 549	12,856 950	0,565			19 285,43
28	9,5	19	23,900 418	13,715 790	0,574			20 573,68
29	10	20	25,037 501	14,578 440	0,582			21 867,66
30	10,5	21	26,171 094	15,444 610	0,590			23 166,92
31	11	22	27,301 455	16,314 040	0,598			24 471,06
32	11,5	23	28,428 790	17,186 500	0,605			25 779,75
33	12	24	29,553 320	18,061 800	0,611			27 092,70
34	12,5	25	30,675 199	18,939 750	0,617			28 409,62
35	13	26	31,794 609	19,820 190	0,623			29 730,28
36	13,5	27	32,911 683	20,702 980	0,629			31 054,47
37	14	28	34,026 569	21,587 970	0,634			32 381,95
38	14,5	29	35,139 366	22,475 050	0,639			33 712,58
39	15	30	36,250 182	23,364 110	0,645			35 046,17
40	15,5	31	37,359 130	24,255 060	0,649			36 382,59
41	16	32	38,466 305	25,147 780	0,654			37 721,67
42	16,5	33	39,571 798	26,042 210	0,654			39 063,32
43	17	34	40,675 641	26,938 270	0,662			40 407,4
44	17,5	35	41,777 972	27,835 880	0,666			41 753,81
45	**18**	**36**	**42,878 793**	**28,734 960**	**0,670**			**43 102,44**

Figure F.4 – Determination of the test termination time

As shown in Figure F.4, termination time is calculated for variation of r and consequently the chi-square functions. In equations shown in row 9 are embedded in row 10. When the ratio of the two chi-square functions reaches a value greater than or equal to the value in cell E5 (0,67 in this example), the closest integer for the number of failures is the number of failures determined as rejection criteria. This number is used for calculation of the test termination time. In order to facilitate selection of the closest integer the computation is made for values of r with a step of 0,5 causing $2r$ to step through all integers. Column H has an embedded formula for the test termination time corresponding to the chi-square function of column G and to the rejection failure number. In the example the test termination time is in bold and is equal to 43 102,44 h at r_0 = 18.

Further derivations of the spreadsheet to obtain the two lines, accept and reject along with the termination time and maximum allowed test failures is shown in Figure F.5.

	L	M	N	O	P	Q	R	S	T
1	A	b	c	Variable	Variable	a+bt	c+bt		
2						=$L$3+$M$3*O5	=$N$3+$M$3*O5		
3	-3,41902	0,000411	2,969362						
4				t(hours)	t/m_0	a+bt	c+bt		
5				0	0	-3,41902	2,969362		
6				400	0,133333	-3,2546	3,133783		
7				800	0,266667	-3,09018	3,298203		
8				1200	0,4	-2,92576	3,462623		
9				1600	0,533333	-2,76134	3,627043		
10				2000	0,666667	-2,59692	3,791463		
11				2400	0,8	-2,4325	3,955884		
12				2800	0,933333	-2,26808	4,120304		
13				3200	1,066667	-2,10366	4,284724		
14				3600	1,2	-1,93924	4,449144		
15				4000	1,333333	-1,77482	4,613565		
90				34000	11,33333	10,5567	16,94508		
91				34400	11,46667	10,72112	17,1095		
92				34800	11,6	10,88554	17,27392		
93				35200	11,73333	11,04996	17,43834		
94				35600	11,86667	11,21438	17,60276		
95				36000	12	11,3788	17,76718		
96				36400	12.13333	11,54322	17,9316	17,9316	
97				36800	12.26667	11,70764		17,9316	
98				37200	12,4	11,87206		17,9316	
99				37600	12,53333	12,03648		17,9316	
100				38000	12,66667	12,2009		17,9316	
101				38400	12,8	12,36532		17,9316	
102				38800	12,93333	12,52974		17,9316	
103				39200	13,06667	12,69416		17,9316	
104				39600	13,2	12,85858		17,9316	
104				40000	13,33333	13,023		17,9316	
106				40400	13,46667	13,18742		17,9316	
107				40800	13,6	13,35184		17,9316	
108				41200	13,73333	13,51626		17,9316	
109				41600	13,86667	13,68068		17,9316	
110				42000	14	13,8451		17,9316	
111				42400	14,13333	14,00952		17,9316	
112				42800	14,26667	14,17394		17,9316	
113				43102	14,36733	14,29808		17,9316	14,29808
114				43102	14,36733	14,29808		17,9316	17,9316

Figure F.5 – Equations for accept and reject line along with the test termination

The upper, reject line is stopped at the time the number of failures reaches the reject number (approximately 18 in this example), This number is pasted in column S all through the reject time (to make the horizontal line in the plot). Column T that contains the number of failures increasing from the accept to reject lines at the test termination time, is completed by using the last number of column Q and the last number of column R for the same termination time to form a vertical test termination line.

Column P of Figure F.5 is the test time measured in m_0 plotted as a function of test time. The accept and reject line along with the test termination time and reject number of failures are shown in Figure F.6.

Figure F.6 shows the plot that is automatically redrawn every time the test parameters are changed (α, β, m_1, and m_0). The only intervention needed from the operator is the addition of the rejection number of failures line, and the test termination line (columns R, S and T in Figure F.5).

Value 17,931 6 that is the number of failures correctly calculated for the corresponding time $\frac{T^*}{m_0}$ of 12,133 33. This number can be substituted in the spreadsheet by the integer 18, or the corresponding test time may be extended by a very small amount such that the calculated number of failures closely approaches 18 so that the reject number of failures appears as an integer.

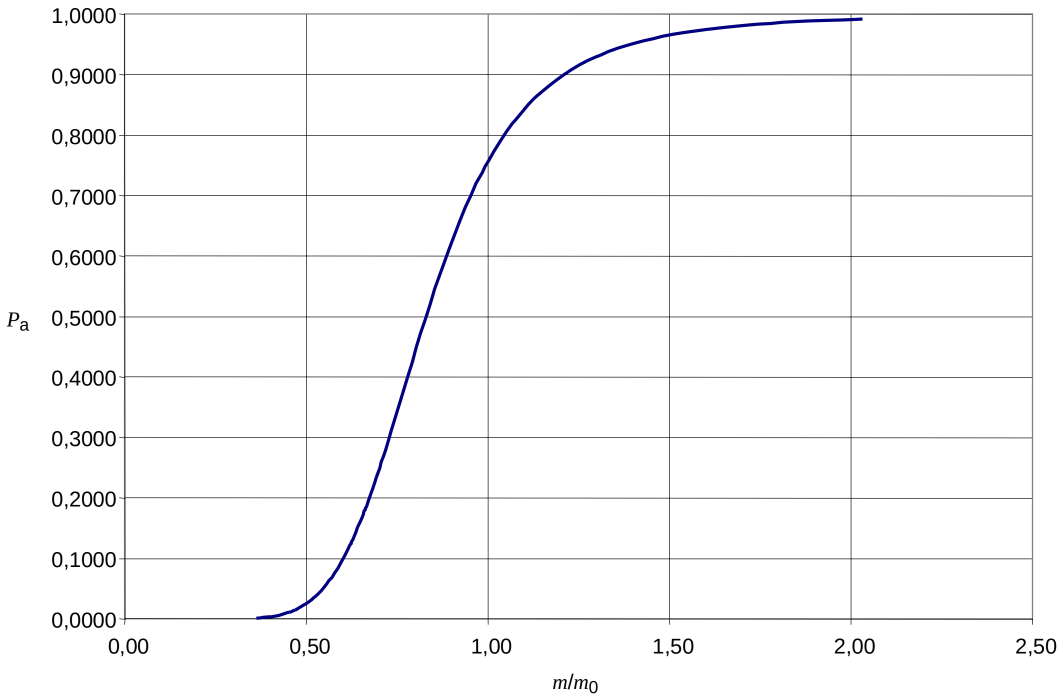

Figure F.6 – SPRT spreadsheet graphing example

The graph in Figure F.5 was drawn by selecting the test times (in m_0) and the values for the accept and reject line along with the reject number of failures and the test termination time. The graph is a scatter graph line.

Once set up with all embedded equations, the graphs can be re-drawn for any required parameters.

A spreadsheet can be set up to automatically draw the OC curves for the given m_0 and m_1 The latter may be adjusted once the curves are completed to change the test time necessary for the required risks. Figure F.7 shows the spreadsheet for the example given in Figure F.6.

F.4 Construction of the OC curves and the expected test time for decision, T_e^*

A spreadsheet can be set up to automatically draw the OC curves for the given test parameters and m_0. Probability of acceptance is given in Annex E in Equations (E.18) and (E.19):

$$P_a(h) = \frac{A^h - 1}{A^h - B^h} \tag{E.18}$$

Constant *h* is found in the following equation:

$$m = m_0 \cdot \frac{D^h - 1}{h \cdot (D - 1)} \tag{E.19}$$

The second equation, (E.19), cannot be solved for *h*, as to then substitute into Equation (E.18) so that the probability of acceptance can be expressed as a function of the true *m.* For that reason, the values are given to the variable *h* in the spreadsheet and then the values of m are calculated, The values are assigned to *h* in such a manner to obtain m in a range from value less than m_1 through values greater than m_0, At the same time, values of $P_a(h)$ are calculated for the same given values of *h*, and then the graph is plotted with the abscissa being the calculated values of m/m_0, and the values of $P_a(h)$ are plotted on the y-axis.

The expected test time T^*_e for decision in measures of *m* is calculated from Equation (E.26) in Annex E:

$$\frac{T_e^*}{m_0} = \frac{E_t}{m_0} = \frac{m}{n \cdot m_0} \cdot E_r = \frac{m}{n \cdot m_0} \cdot m_0 \cdot \frac{P_a[\ln(A) - \ln(B)] - \ln(A)}{m \cdot (D-1) - m_0 \cdot \ln(D)}$$

$$\frac{T_e^*}{m_0} = m \cdot \frac{P_a[\ln(A) - \ln(B)] - \ln(A)}{m \cdot (D-1) - m_0 \cdot \ln(D)} \tag{E.26}$$

Figure F.7 shows the spreadsheet for the example shown in Figures F.1 through F.6 with embedded equations that apply to the parameters below them. Row 3 gives the equations as they are embedded in row 5.

The corresponding values for m_0, A, B, and *D* are copied from their place in the spreadsheet. The symbol $ in this spreadsheet type denotes that the cell value is fixed, and would not change with another row or column.

	X	Y	Z	AA	AB	AC	AD	AE
1						*n*		
2						1		
3	**Chosen values**	**=(($E$3^X5-1)/(X5*($E$3-1)))*$F$3**		**=($J$4^X5-1)/ ($J$4^X5-$K$4^X5)**			**=(Y5/$AC$2)* AE5/$F$3**	**=$F$3*(AA5* (LN($J$4)-LN($K$4))-LN($J$4))/(Y5* ($E$3-1)-$F$3*LN($E$3))**
4	h	m	m/m_0	P_a		m/m_0	T_e*/m_0	E_r
5	-5,0	1041,98	0,35	0,0010		0,35	1,80	5,18
6	-4,7	1086,74	0,36	0,0015		0,36	1,94	5,35
7	-4,4	1134,60	0,38	0,0022		0,38	2,09	5,54
8	-4,1	1185,83	0,40	0,0034		0,40	2,27	5,75
9	-3,8	1240,71	0,41	0,0051		0,41	2,48	5,99
10	-3,5	1299,56	0,43	0,0077		0,43	2,72	6,27
11	-3,2	1362,72	0,45	0,0116		0,45	2,99	6,58
12	-2,9	1430,57	0,48	0,0174		0,48	3,31	6,94
13	-2,6	1503,54	0,50	0,0260		0,50	3,68	7,34
14	-2,3	1582,07	0,53	0,0387		0,53	4,10	7,78
15	-2,0	1666,67	0,56	0,0572		0,56	4,59	8,27
31	2,8	4525,96	1,51	0,9663		1,51	5,62	3,72
32	3,1	4867,08	1,62	0,9764		1,62	5,30	3,27
33	3,4	5239,88	1,75	0,9835		1,75	5,02	2,87
34	3,7	5647,59	1,88	0,9884		1,88	4,77	2,53
35	4,0	6093,75	2,03	0,9919		2,03	4,55	2,24
36	4,3	6582,31	2,19	0,9944		2,19	4,35	1,98
37	4,6	7117,61	2,37	0,9961		2,37	4,18	1,76
38	4,9	7704,50	2,57	0,9973		2,57	4,03	1,57
39	5,2	8348,32	2,78	0,9981		2,78	3,90	1,40
40	5,5	9054,99	3,02	0,9987		3,02	3,78	1,25
41	5,8	9831,09	3,28	0,9991		3,28	3,68	1,12
42	6,1	10683,90	3,56	0,9994		3,56	3,59	1,01
43	6,4	11621,51	3,87	0,9995		3,87	3,50	0,90
44	6,7	12652,87	4,22	0,9997		4,22	3,43	0,81
45	7,0	13787,95	4,60	0,9998		4,60	3,37	0,73
46	7,3	15037,77	5,01	0,9998		5,01	3,31	0,66
47	7,6	16414,60	5,47	0,9999		5,47	3,25	0,59
48	7,9	17932,05	5,98	0,9999		5,98	3,21	0,54
49	8,2	19605,24	6,54	0,9999		6,54	3,17	0,48
50	8,5	21450,97	7,15	1,0000		7,15	3,13	0,44

Figure F.7 – Spreadsheet set-up for construction of the OC curves for the SPRT

The computation ends when AA reaches 1.

Selecting Z5:AA50, and plotting a scatter line, OC curve for the probability of acceptance P_a is plotted (shown in Figure F.8).

The data source AC5:AD50 produces a graph for the expected test duration for reaching a decision, measured in units m_0 (shown in Figure F.8).

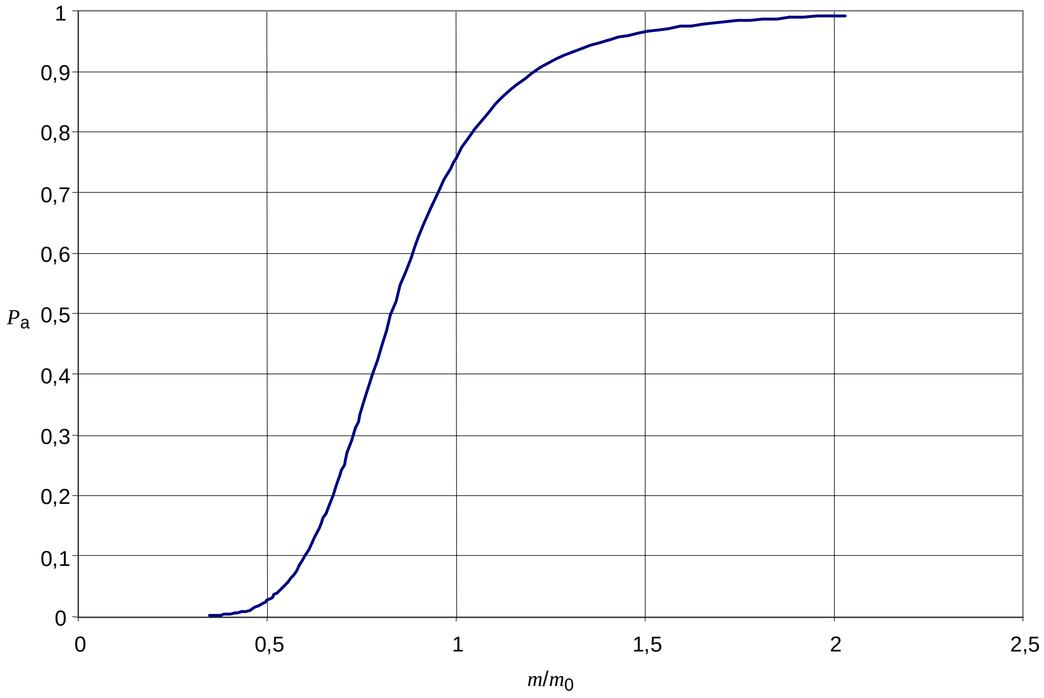

Figure F.8 – OC curve for probability of acceptance, P_a

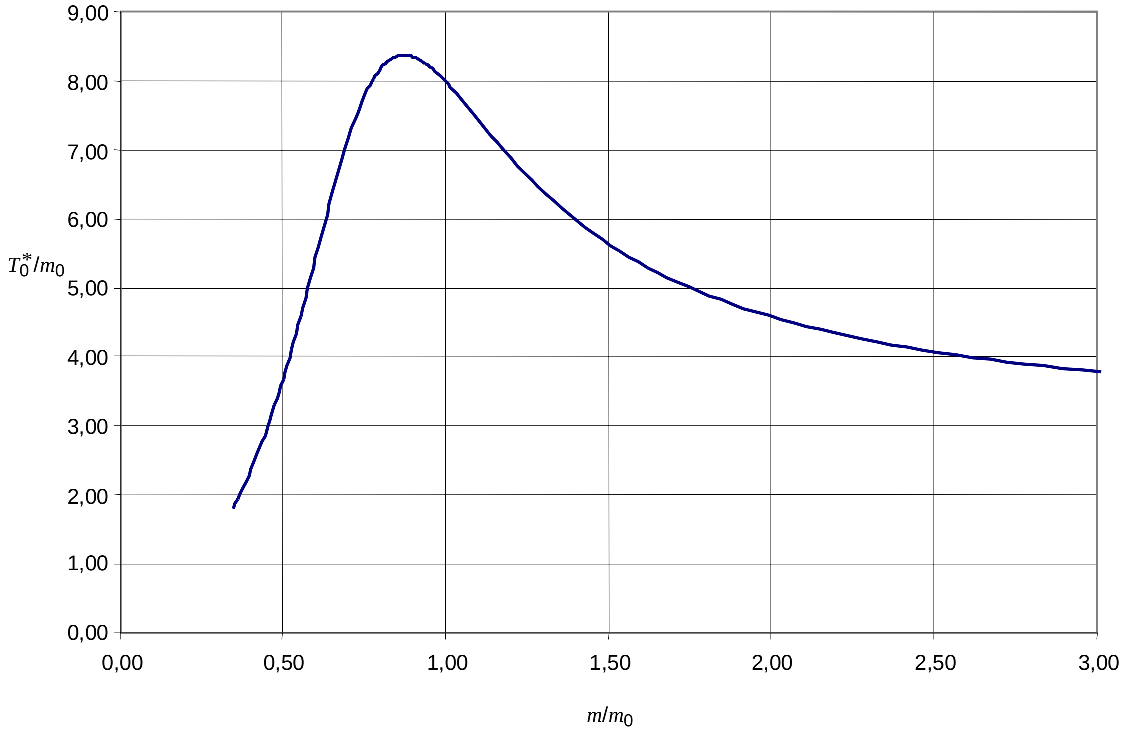

Figure F.9 – Expected test time for making a decision

F.5 Step-by-step procedure

To calculate a test plan follow the following steps:

Step 1: Key in $\alpha, \beta, D, m_0, m_1$ as specified

Step 2: Find in column E the first value larger than or equal to value in E5. Read in same line T_t^* in column H and r_0 in column A.

Step 3: Find value closest to T_t^* in column O and type in the value found in step 2.

Step 4: Copy T_t^* into the rest of column O to the bottom of the sheet.

Step 5: Find value closest to r_0 in column R. Copy that value to column S from the same line to the bottom of the sheet. Delete values in column R below this line.

Step 6: For the line with the T_t^* found in step 5 in column O copy value in same line from column Q to the same line in column T. If there are values from a previous calculation in column S and T they have to be deleted.

Step 7: Graphs are automatically updated.

Annex G
(informative)

Examples and mathematical references for fixed duration time/failure terminated test plans

NOTE This annex uses the symbols listed in 3.2.2.

G.1 Mathematical references

G.1.1 General

NOTE See Clause 7.

There are four possible considerations that are applicable to the fixed duration tests:

a) time terminated tests with replacement;
b) time terminated tests without replacement;
c) failure terminated tests with replacement;
d) failure terminated tests without replacement.

Failure rate and/or MTBF calculations are done as shown in 3.2.2.

When fixed duration, time terminated tests are conducted with replacement, the test termination time, T_t^* and the acceptable number of failures *c* is determined based on the following two equations:

$$\beta = \sum_{k=0}^{a} \frac{\left(\frac{T}{m_1}\right)^k \cdot e^{-\frac{T}{m_1}}}{k!}$$
$$1-\alpha - \sum_{k=0}^{a} \frac{\left(\frac{T}{m_0}\right)^k \cdot e^{-\frac{T}{m_0}}}{k!} \qquad \text{(G.1)}$$

The Equations (G.1) can be entered into a spreadsheet to determine time as well as the number of acceptable failures based on the required m_0 discrimination ratio, and producer's and customer's risk, α and β. An example is shown in the Informative Annex F.

The OC curve is given by plotting the probability of acceptance, P_a (in a test of predetermined test duration T^*) against the quantity *m*, where:

$$P_a = e^{-\frac{T^*}{m}} \sum_{k=0}^{c} \frac{\left(\frac{T^*}{m}\right)^k}{k!} \qquad \text{(G.2)}$$

G.2 Examples on design of fixed duration failure terminated test plans

G.2.1 Example 1

Specified: A repaired equipment has a target MTBF = $m_0 \geq$ 1 500 h. One item is field-tested. The producer accepts α = 5 % risk to be rejected compliance, even if the target has been reached. The consumer accepts β = 5 % risk to accept compliance, even if the true m = 750 h, thus D = 1 500/750 = 2.

To be derived: Time/failure terminated test plan with accumulated test time for termination, T_e^*, and the acceptable number of failures during the test, *c.*

Procedures: Test plan B.2 states that $\frac{T^*}{m_0}$ = 15,71 and c = 22. The test time is thus

$$T^* = 15{,}7\ m_0 = 15{,}71 \times 1\ 500 = 23\ 550 \text{ h} = 2{,}7 \text{ years}$$

during which 22 or less failures are acceptable for compliance.

NOTE If the test time is considered too long, due to time scheduling, etc., the number of items under test may be increased, or D and/or the risks increased, A suitable and "balanced" test plan may also be designed according to 8.2, where the test time can be chosen independently of m_0 Combined test plans (see Annex D) should also be considered.

G.2.2 Example 2

Specified: The same data as example D.1: n = 500, $\lambda_0 = 900 \times 10^{-9}$ h^{-1} and $\alpha = \beta$ = 10 % and D = 3.

To be derived: Time/failure terminated test plan with accumulated test time for termination, T_t^*, and the acceptable number of failures during the test, *c.*

Procedures: Test plan B.7 states that $\frac{T^*}{m_0}$ = 3,1 and c = 5. According to 3.2.2, $m_0 = \frac{1}{\lambda_0}$ and the test time is thus $T^* = \frac{3{,}1}{\lambda_0} = \frac{3{,}1}{900 \times 10^{-9}}$ = 3 44$\times 10^3$ h, and $t_* = \frac{T^*}{n}$ = 6 889 h = 0,8 years, during which five or less failures are acceptable for compliance.

NOTE Compared with the example in Clause E.2 using the sequential test plan A.3, it is seen that the test time for the time/failure terminated test plan is slightly shorter than the maximum test time for the sequential test plan. For very good and very bad items, the test time for the sequential test plan may be significantly shorter; however, it is more complicated to schedule and administrate.

Annex H
(informative)

Design of fixed duration time/failure terminated test plans using a spreadsheet program

NOTE 1 See Clause 7.

NOTE 2 This Annex uses the symbols listed in 3.2.2 with the exception that T^* is called T due to limitation of the equation editor in the spreadsheet program.

The equations (G.1) can be entered into a spreadsheet to determine time as well as the number of acceptable failures based on m_0 , m_1 , discrimination ratio, D and producer's and customer's risk, α and β as shown in Figures H.1 to H.4.

In Figure H.1, row 7, columns B to L contain the equation above this part of the spreadsheet (β from Equation (G.2)), while columns M to W contain embedded equation shown above those cells (equation for 1 - α from Equation (G.1)).

Row 5 points out to those cells in row 7, and is related to the embedded equations shown in Figure H.2. Each of those equations is entered into the appropriate cell of the row 7, and then this row is copied and pasted down the spreadsheet for the length of time estimated for the test (shown in column Y).

Columns Z and AA contain times divided by m_1 and m_0 to express the time in terms of m and those are used in the embedded equations.

The example in Figure F.1 shows the maximum acceptable number of failures up to 10. This number, if needed, can be extended and the embedded equations have to be adjusted accordingly.

The equations are shown in quotation marks as this was the way to make them visible in the spreadsheet. When entered into the respective cells, the quotation marks have to be omitted.

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB
1		α	$1-\alpha$	β	m_1	m_0		D																			
2		0,2	0,8	0,2	500	1000		2																			
3						$exp\left(-\frac{T}{m_1}\right)\cdot\sum_{k=0}^{c}\frac{\left(\frac{T}{m_1}\right)^k}{k!}$											$exp\left(-\frac{T}{m_0}\right)\cdot\sum_{k=0}^{c}\frac{\left(\frac{T}{m_0}\right)^k}{k!}$					$exp\left(-\frac{T}{m_1}\right)$				$exp\left(-\frac{T}{m_0}\right)$	
4						β											$1-\alpha$										
	c=0	c=1	c=2	c=3	c=4	c=5	c=6	c=7	c=8	c=9	c=10	c=0	c=1	c=2	c=3	c=4	c=5	c=6	c=7	c=8	c=9	c=10					
5																							=EXP(-Z7)	T	T/m_1	T/m_0	=EXP(-AA7)
7	0,368	0,736	0,920	0,981	0,996	0,999	1,000	1,000	1,000	1,000	1,000	0,607	0,910	0,986	0,998	1,000	1,000	1,000	1,000	1,000	1,000	1,000	0,368	500	1,000	0,500	0,607
8	0,333	0,699	0,900	0,974	0,995	0,999	1,000	1,000	1,000	1,000	1,000	0,577	0,894	0,982	0,998	1,000	1,000	1,000	1,000	1,000	1,000	1,000	0,333	550	1,100	0,550	0,577
9	0,301	0,663	0,879	0,966	0,992	0,998	1,000	1,000	1,000	1,000	1,000	0,549	0,878	0,977	0,997	1,000	1,000	1,000	1,000	1,000	1,000	1,000	0,301	600	1,200	0,600	0,549
10	0,273	0,627	0,857	0,957	0,989	0,998	1,000	1,000	1,000	1,000	1,000	0,522	0,861	0,972	0,996	0,999	1,000	1,000	1,000	1,000	1,000	1,000	0,273	650	1,300	0,650	0,522
11	0,247	0,592	0,833	0,946	0,986	0,997	0,999	1,000	1,000	1,000	1,000	0,497	0,844	0,966	0,994	0,999	1,000	1,000	1,000	1,000	1,000	1,000	0,247	700	1,400	0,700	0,497
12	0,223	0,558	0,809	0,934	0,981	0,996	0,999	1,000	1,000	1,000	1,000	0,472	0,827	0,959	0,993	0,999	1,000	1,000	1,000	1,000	1,000	1,000	0,223	750	1,500	0,750	0,472
13	0,202	0,525	0,783	0,921	0,976	0,994	0,999	1,000	1,000	1,000	1,000	0,449	0,809	0,953	0,991	0,999	1,000	1,000	1,000	1,000	1,000	1,000	0,202	800	1,600	0,800	0,449
14	0,183	0,493	0,757	0,907	0,970	0,992	0,998	1,000	1,000	1,000	1,000	0,427	0,791	0,945	0,989	0,998	1,000	1,000	1,000	1,000	1,000	1,000	0,183	850	1,700	0,850	0,427
15	0,165	0,463	0,731	0,891	0,964	0,990	0,997	0,999	1,000	1,000	1,000	0,407	0,772	0,937	0,987	0,998	1,000	1,000	1,000	1,000	1,000	1,000	0,165	900	1,800	0,900	0,407
16	0,150	0,434	0,704	0,875	0,956	0,987	0,997	0,999	1,000	1,000	1,000	0,387	0,754	0,929	0,984	0,997	1,000	1,000	1,000	1,000	1,000	1,000	0,150	950	1,900	0,950	0,387
17	0,135	0,406	0,677	0,857	0,947	0,983	0,995	0,999	1,000	1,000	1,000	0,368	0,736	0,920	0,981	0,996	0,999	1,000	1,000	1,000	1,000	1,000	0,135	1000	2,000	1,000	0,368
18	0,122	0,380	0,650	0,839	0,938	0,980	0,994	0,999	1,000	1,000	1,000	0,350	0,717	0,910	0,978	0,996	0,999	1,000	1,000	1,000	1,000	1,000	0,122	1050	2,100	1,050	0,350
19	0,111	0,355	0,623	0,819	0,928	0,975	0,993	0,998	1,000	1,000	1,000	0,333	0,699	0,900	0,974	0,995	0,999	1,000	1,000	1,000	1,000	1,000	0,111	1100	2,200	1,100	0,333
20	0,100	0,331	0,596	0,799	0,916	0,970	0,991	0,997	0,999	1,000	1,000	0,317	0,681	0,890	0,970	0,993	0,999	1,000	1,000	1,000	1,000	1,000	0,100	1150	2,300	1,150	0,317

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70	0,001	0,006	0,024	0,067	0,147	0,264	0,406	0,554	0,689	0,799	0,879	0,026	0,121	0,294	0,505	0,697	0,837	0,923	0,967	0,987	0,996	0,999	0,001	3650	7,300	3,650	0,026
71	0,001	0,005	0,022	0,063	0,140	0,253	0,392	0,539	0,676	0,788	0,871	0,025	0,116	0,285	0,494	0,687	0,830	0,918	0,965	0,986	0,995	0,998	0,001	3700	7,400	3,700	0,025
72	0,001	0,005	0,020	0,059	0,132	0,241	0,378	0,525	0,662	0,776	0,862	0,024	0,112	0,277	0,484	0,678	0,823	0,914	0,962	0,985	0,995	0,998	0,001	3750	7,500	3,750	0,024
73	0,001	0,004	0,019	0,055	0,125	0,231	0,365	0,510	0,648	0,765	0,854	0,022	0,107	0,269	0,473	0,668	0,816	0,909	0,960	0,984	0,994	0,998	0,001	3800	7,600	3,800	0,022
74	0,000	0,004	0,017	0,052	0,118	0,220	0,351	0,496	0,634	0,753	0,845	0,021	0,103	0,261	0,463	0,658	0,808	0,904	0,957	0,983	0,994	0,998	0,000	3850	7,700	3,850	0,021
75	0,000	0,004	0,016	0,048	0,112	0,210	0,338	0,481	0,620	0,741	0,835	0,020	0,099	0,253	0,453	0,648	0,801	0,899	0,955	0,981	0,993	0,998	0,000	3900	7,800	3,900	0,020
76	0,000	0,003	0,015	0,045	0,106	0,201	0,326	0,467	0,607	0,729	0,826	0,019	0,095	0,246	0,443	0,639	0,793	0,894	0,952	0,980	0,993	0,997	0,000	3950	7,900	3,950	0,019
77	0,000	0,003	0,014	0,042	0,100	0,191	0,313	0,453	0,593	0,717	0,816	0,018	0,092	0,238	0,433	0,629	0,785	0,889	0,949	0,979	0,992	0,997	0,000	4000	8,000	4,000	0,018
78	0,000	0,003	0,013	0,040	0,094	0,182	0,301	0,439	0,579	0,704	0,806	0,017	0,088	0,231	0,424	0,619	0,777	0,884	0,946	0,977	0,991	0,997	0,000	4050	8,100	4,050	0,017
79	0,000	0,003	0,012	0,037	0,089	0,174	0,290	0,425	0,565	0,692	0,796	0,017	0,085	0,224	0,414	0,609	0,769	0,879	0,943	0,976	0,990	0,997	0,000	4100	8,200	4,100	0,017
80	0,000	0,002	0,011	0,035	0,084	0,165	0,278	0,412	0,551	0,679	0,785	0,016	0,081	0,217	0,405	0,600	0,761	0,873	0,939	0,974	0,990	0,996	0,000	4150	8,300	4,150	0,016

NOTE Time T in the equations shown in the spreadsheet has the symbol T instead of T^*. This is because of the limitation of the equation editor, T denotes T^* in the spreadsheet.

Figure H.1 – Set-up of the spreadsheet with embedded equations – Example

B7	"=((Z7^$B$5)/1)*X7"
C7	"=(((Z7^$C$5)/$C$5)+B7/X7)*X7"
D7	"=((Z7^$D$5)/(PRODUCT($C$5:$D$5))+C7/X7)*X7"
E7	"=((Z7^$E$5)/(PRODUCT($C$5:$E$5))+D7/X7)*X7"
F7	"=(((Z7)^$F$5)/(PRODUCT($C$5:$F$5))+E7/X7)*X7"
G7	"=(((Z7)^$G$5)/(PRODUCT($C$5:$G$5))+F7/X7)*X7"
H7	"=(((Z7)^$H$5)/(PRODUCT($C$5:$H$5))+G7/X7)*X7"
I7	"=(((Z7)^$I$5)/(PRODUCT($C$5:$I$5))+H7/X7)*X7"
J7	"=(((Z7)^$J$5)/(PRODUCT($C$5:$J$5))+I7/X7)*X7"
K7	"=(((Z7)^$K$5)/(PRODUCT($C$5:$K$5))+J7/X7)*X7"
L7	"=(((Z7)^$L$5)/(PRODUCT($C$5:$L$5))+K7/X7)*X7"
M7	"=((AA7^$M$5)/1)*AB7"
N7	"=(((AA7^$N$5)/$N$5)+M7/AB7)*AB7"
O7	"=((AA7^$O$5)/(PRODUCT($N$5:$O$5))+N7/AB7)*AB7"
P7	"=(((AA7)^$P$5)/(PRODUCT($N$5:$P$5))+O7/AB7)*AB7"
Q7	"=(((AA7)^$Q$5)/(PRODUCT($N$5:$Q$5))+P7/AB7)*AB7"
R7	"=(((AA7)^$R$5)/(PRODUCT($N$5:$R$5))+Q7/AB7)*AB7"
S7	"=(((AA7)^$S$5)/(PRODUCT($N$5:$S$5))+R7/AB7)*AB7"
T7	"=(((AA7)^$T$5)/(PRODUCT($N$5:$T$5))+S7/AB7)*AB7"
U7	"=(((AA7)^$U$5)/(PRODUCT($N$5:$U$5))+T7/AB7)*AB7"
V7	"=(((AA7)^$V$5)/(PRODUCT($N$5:$V$5))+U7/AB7)*AB7"
W7	"=(((AA7)^$W$5)/(PRODUCT($N$5:$W$5))+V7/AB7)*AB7"

Figure H.2 – Equations embedded into the spreadsheet shown in Figure H.1

To achieve the goal of this process, determine the test duration and the acceptable number of failures, the spreadsheet is examined to find the test termination time T_e^*. In this way, requirements for producer and consumer's risk are satisfied at the same time. Figure H.3 shows how this is accomplished, using the same example as for Figures H.1 and H.2

$$\beta = \sum_{k=0}^{a} \frac{\left(\frac{T}{m_1}\right)^k \cdot e^{-\frac{T}{m_1}}}{k!}$$

$$1-\alpha - \sum_{k=0}^{a} \frac{\left(\frac{T}{m_0}\right)^k \cdot e^{-\frac{T}{m_0}}}{k!} \tag{G.1}$$

In the spreadsheet of Figure H.1 the number of acceptable failures is found by looking for the closest value that satisfies both equations above. It is important to be conservative with the choice of value for consumer's risk; this value should be equal to or less than that given by the test plan. The value of producer's risk α should be close to the value given in the test plan. Both selected values shall produce the same acceptable number of failures. The time that corresponds to those two numbers is the test termination time T_t^*.

In Figure H.1, the numbers closest to the producer's and customer's risk that correspond to the same number of failures is 0,201 (for β), being the required consumer's risk, and 0,793 ($1 - \alpha$), allowing for a somewhat larger producer's risk (the test plan for both risks should be 0,2). The test time corresponding to these values is T = 3 950 h. In the example of Figure H.3, it can be seen that the time corresponding coincidentally to both the requirement for α and the requirement for β is 3 950 h or 7,9 times m_1 and 3,9 times m_0.

Step by step procedure:

Step 1: Type in α, β, m_0, m_1 and *D* as specified as shown in Figure H.1.

Step 2: Type in the equations from Figure H.2.

Step 3: Update data in step 1 if required to design a new test plan.

Step 4: For number of failures *c* find, in columns B through L, the row where the value is close to β.

Step 5: For number of failures *c* find, in columns M through W, the row where the value is close to $1-\alpha$.

Step 6: Check if the row number found in step 4 is the same as the row number found in step 5 for the same *c*.

If yes: then *c* is the acceptable number of failures for the test plan.

Find, in column AA, in the same row, the termination test time T_t^* / m_0.

If no: go to step 7.

Step 7: Repeat steps 4 through 6 with a new value of *c* until a suitable test plan is found.

Step 8: Once a test plan has been found, go to step 3 to design another test plan, if required.

The spreadsheet for plotting of the OC curve is shown in Figure H.3.

	AE	AF	AG	AH	AI	AJ	AK
4							$\sum_{k=0}^{5} \frac{\left(\frac{T}{m}\right)^k}{k!}$
5	*T*	**3 950**	*m*	m/m_0	*P*(*m*)	exp(-*T*/*m*)	
6			1	0,001	0	0	8,02332E+15
7			100	0,1	6,40087E-12	7,00435E-18	913841,544
8			200	0,2	8,70204E-05	2,64657E-09	32880,41747
9			300	0,3	0,009626203	1,91333E-06	5031,131547
10			400	0,4	0,071965387	5,14449E-05	1398,883901
11			500	0,5	0,200569118	0,000370744	540,9915374
12			600	0,6	0,357043975	0,001383231	258,1232042
13			700	0,7	0,504604639	0,003542732	142,4337663
14			800	0,8	0,626925929	0,007172507	87,40680312
15			900	0,9	0,721784843	0,012414515	58,1403958
16			1000	1	0,792895407	0,019254702	41,1793138
17			1100	1,1	0,845367954	0,027573252	30,65898579
18			1200	1,2	0,883877046	0,037191811	23,765367
19			1300	1,3	0,912154808	0,047908538	19,03950404
20			1400	1,4	0,933005094	0,059520852	15,67526438
21			1500	1,5	0,948473775	0,071838601	13,20284312
22			1600	1,6	0,960033944	0,084690656	11,3357717
23			1700	1,7	0,968741487	0,097927349	9,892450849
24			1800	1,8	0,975353686	0,111420445	8,753812547
25			1900	1,9	0,980415627	0,125061787	7,839450014
26			2000	2	0,984321804	0,138761312	7,093632861
27			2100	2,1	0,987359585	0,152444851	6,476831318
28			2200	2,2	0,989739798	0,166051957	5,960422361
29			2300	2,3	0,991618264	0,179533887	5,523293018
30			2400	2,4	0,993111017	0,192851786	5,149607571
31			2500	2,5	0,994305111	0,205975098	4,827307377
32			2600	2,6	0,995266335	0,218880192	4,547082713
33			2700	2,7	0,996044769	0,231549186	4,301655243
34			2800	2,8	0,996678794	0,243968957	4,085268902
35			2900	2,9	0,997198033	0,256130304	3,89332312
36			3000	3	0,997625488	0,268027239	3,72210486

Figure H.3 – OC curve for the time/failure terminated fixed duration test

In Figure H.3 the equation to be written in column AK of the spreadsheet is as follows:

AK6 "=1+($AE$5/AA37)+(1/2)*($AE$5/AA37)^2+(1/6)*($AE$5/AA37)^3+(1/24)*($AE$5/AA37)^4+(1/120)*($AE$5/AA37)^5"

Enter the following In cell AJ6:

AJ6: “=exp(–$AE$5/AF6)"

The probability of acceptance is then entered into AI6 as follows:

“=AJ6*AK6”

AH is entered as follows:

“=AG6/$G$2”

All the entries described above shall be made without the quotation marks. Figure H.4 shows the OC curve plotted for the example in Figures H.1 to H.3. AG is plotted as a function of AH.

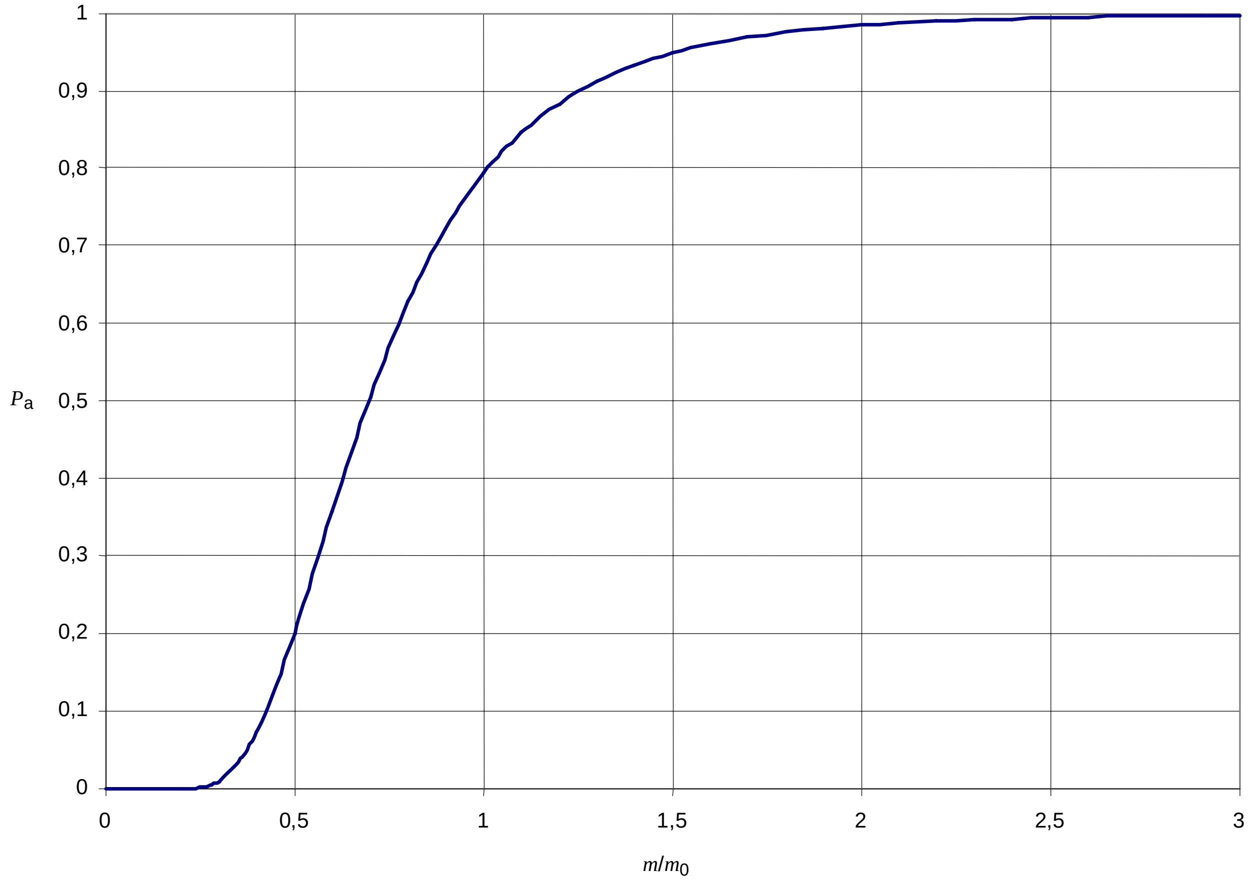


Figure H.4 – OC curve plotted from the spreadsheet calculations

Annex I
(informative)

Examples and mathematical references for the design of alternative time/failure terminated test plans

NOTE 1 See Clause 8.

NOTE 2 This annex shows detailed examples referring to the procedures in Clause 8.

I.1 Symbols

In addition to the symbols given in 3.2.2 the following symbols are used in this annex:

c_t	tentative value for rounding c to the nearest integer
D'	true discrimination ratio
$f(P_a, c)$	a function of P_a and c
$P(c)$	the cumulative Poisson distribution
$p(c)$	probability density function of the Poisson distribution
t^*	calendar time
u_γ	γ – fractile of the cumulative normal distribution $\gamma = \Phi(u)$
$\Phi(u)$	cumulative normal distribution
γ	fractile of the distribution
μ_{0t}	a tentative value for rounding μ to the nearest integer
μ_1	value of μ used to determine D'

I.2 Examples

I.2.1 Example 1

Specified: λ_0 = 0,025 failures per year, $\alpha = \beta$ = 5 %, D = 2.

To be derived: c, T_t^*

Procedures: μ_0 = 15,8 for D = 2 is found from the $\alpha = \beta$ = 5 % curve in Figure C.3.

$\Delta\mu_0$ = 6,3 for μ_0 = 15,8 is found from the $\alpha = \beta$ = 5 % curve in Figure C.4.

Thus c = (15,8 + 6,3) rounded to 22, Then $T_t^* = \frac{\mu_0}{\lambda_0} = \frac{15{,}8}{0{,}025} =$ 632 component years.

For a specified test time t_t^* = 1 year, n = 632 components.

For a specified number of components n = 1 000 follows $t^* = \frac{T_t^*}{n} = \frac{632}{1\ 000} \approx 5\ 500$ h.

I.2.2 Example 2

Specified: λ_0 = 900 failures per 10^9 h, $\alpha = \beta$ = 10 %, $D \approx 3$

To be derived: c, T_t^*, D'

Procedures: Identify the point on the dashed $\alpha = \beta$ = 10 % curve in Figure C.1 for which D = 3, Follow the dashed curve to the intersection with the nearest c curve.

Thus c = 5 is determined, giving μ_0 = 3,15 and D' = 2,95.

Then $T_t^* = \frac{\mu_0}{\lambda_0} = \frac{3{,}15}{900 \times 10^{-9}} = 3{,}50 \times 10^6$ component hours.

For a specified test time t_t^* = 0,5 year = 4 380 h follows $n = \frac{T_t^*}{t_t^*} = \frac{3{,}50 \times 10^6}{4\ 380} = 799$ components.

For a specified number of components n = 500 follows $t_t^* = \frac{T_t^*}{n} = \frac{3{,}50 \times 10^6}{500} =$ 7 000 h.

NOTE In this example, Figure C.1 should be used because the indicated point would appear just on the lower limit of Figures C.3 and C.4. In the overlapping range of the figures, Figure C.1 is the most accurate. In this example, Figure C.3 gives μ_0 = 3,05 whereas Figure C.1 gives μ_0 = 3,15. Both Figures C.1 and C.4 give c = 5.

I.3 Mathematical procedures

I.3.1 General

The procedures and formulae in this annex may be used to compute the test plan parameters instead of reading the values from the figures.

$$p(c) = \frac{\mu^c e^{-\mu}}{c!} \tag{I.1}$$

$$P(c) = \sum_{i=0}^{c} p(i) = e^{-\mu} \sum_{i=0}^{c} \frac{\mu^i}{i!} \tag{I.2}$$

NOTE In this annex, probabilities are not given as percentages.

I.3.2 Computations

I.3.2.1 Determination of *c* and *D* for $\mu_0 \leq 5$

This case relates to Figure C.1.

Specified: μ_0 , $\alpha = \beta$

To be derived: *D*, *c*

Procedure:

Step 1

Compute a tentative value, c_t for *c* rounding to the nearest integer:

$$c_t = \left[\mu_0 + u_{1-\alpha}\sqrt{\mu_0} - 0{,}5 + \frac{u_{1-\alpha}^2}{8} \right] \text{ (rounded)} \quad \text{(I.3)}$$

where $u_{1-\alpha} = u_\gamma$ may be taken from Table I.2.

Step 2

Compute the α' value from α' = 1 – *P*(*c*) according to Equation (G.1) for $\mu = \mu_0$ utilizing the recursive relationship

$$p(i+1) = p(i)\frac{\mu}{i+1} \quad \text{(I.4)}$$

where $p(0) = e^{-\mu}$

Step 3

Compare α' with α; if they are not reasonably equal, then increase or decrease c_t by 1 and repeat step 2.

Step 4

Let $\beta' = \alpha'$, and compute μ_1 by iteration using Equations (I.1) and (I.2) in such a manner that *P*(*c*) converges to α' and μ to μ_1 .

Step 5

Find the *D*′ value from $D' = \frac{\mu_1}{\mu_0}$.

I.3.2.2 Determination of OC curves

This case relates to Figure C.2 (and Figure C.1).

Specified: *c*

To be derived: Probability of acceptance $P_a = P_a(\mu, c)$ as a function of μ for fixed *c.*

Procedures

Step 1a)

Compute $P_a(\mu, c) = P(c)$ for a suitable range of values of μ using Equations (I.1) and (I.5).

Step 1b)

If $c \geq 3$, P_a may be computed from:

$P_a = \gamma = \Phi(u_\gamma)$

using the approximation

$$u_\gamma \approx \sqrt{9(c+1)} \times \left[\sqrt[3]{\frac{\mu}{c+1}} - 1 + \frac{1}{9(c+1)} \right] \qquad \text{(I.5)}$$

Step 1c)

If $c \geq 9$, P_a may be computed from:

$P_a = \gamma = \Phi(u_\gamma)$

using the simpler approximation:

$$u_\gamma \approx 2\left[2\sqrt{\mu} - \sqrt{2(\mu + c + 0{,}5)}\right] \qquad \text{(I.6)}$$

NOTE Equations (I.5) and (I.6) may be used to find α' and β', considering that for $\gamma > 0{,}5$: $\alpha' = 1-\gamma$, and for $\gamma < 0{,}5$: $\beta' = \gamma$

I.3.2.3 Determination of μ from OC curves

This case relates to Figure C.2.

In some cases, it can be useful to determine values from the OC-curve to obtain for example:

– a starting value for iteration (see I.3.2.1, step 4);
– the first and last μ values for an exact OC curve (see I.3.2.2 step 1);
– the (approximate) μ_1 value for determination of *D* for fixed γ values.

Specified: c

To be derived: Expected number of failures $\mu = f(P_a, c)$, as a function of P_a for fixed c.

Procedures:

Step 1a)

Compute μ for a desired value of $P_a = P(c)$ by iteration, using Equations (I.1) and observing that $\gamma = P_a = \Phi(u_\gamma)$

Step 1b)

If $c \geq 3$, μ may be computed using the approximation:

$$\mu \approx (c+1) \times \left[1 - \frac{1}{9(c+1)} + u_\gamma \sqrt{\frac{1}{9(c+1)}}\right]^3 \qquad \text{(I.7)}$$

Step 1c)

If $c \geq 9$, μ may be computed using the simpler approximation:

$$\mu \approx c + 0{,}5 + 3\frac{u_\gamma^2}{8} + u_\gamma \sqrt{c + 0{,}5 + \frac{u_\gamma^2}{8}} \qquad \text{(I.8)}$$

I.3.2.4 Determination of *D* and c for $\mu_0 > 5$

This case relates to C.1and Figures C.3 and C.4.

Specified: μ_0 , $\alpha = \beta$

To be derived: D, c

Procedures:

Step 1

Compute the c value using the approximate formula (I.3).

Step 2

Compute the D value from the approximate formula:

$$D \approx \left[1 + \frac{u_{1-\alpha}}{\sqrt{\mu_0}}\right]^2 \qquad \text{(I.9)}$$

Step 3 (optional)

Compute $\alpha' = \beta'$, for $\mu = \mu_0$ using one of the following alternatives:

– Equations (I.1) and (I.4); or
– Equation (I.5); or
– Equation (I.6)

Step 4 (optional)

Compute the D' value, either according to I.3.2.1, steps 4 and 5, or using the approximate formulae I.7 or I.8.

I.3.2.5 Determination of *c* and $\alpha = \beta$ or $\alpha' = \beta'$

This case relates to Clause C.2.

Specified: μ_0, D

To be derived: c, α, β

Procedures

Step 1

Compute a tentative $\gamma_t = \alpha = \beta$ from:

$$\gamma_t = \Phi(u_{\gamma_t})$$

where

$$u_{\gamma_t} = \sqrt{\mu_0} \times (\sqrt{D} - 1) \qquad \text{(I.10)}$$

Step 2

Compute a tentative c value, c_t, from Equation (I.3).

Step 3

If $\mu_{0t} \leq 5$, check c_t, D', α' and β' using equation (I.1). If necessary, increase or decrease c accordingly.

Formulae (I.5) and (I.6) may also be used for determination of α' and β' within their valid range.

I.3.2.6 Determination of *c* and *n* or t_t^*

This case relates to Clause C.3.

Specified: λ_0, D, $\alpha = \beta$, and t_t^*, or n

To be derived: c, $\mu_0 = \lambda_0 n t_t^*$, n or t_t^*

Procedures

Step 1

Compute a tentative μ_0 value μ_{0t} from:

$$\mu_{0t} \approx \left[\frac{u_{1-\alpha}}{\sqrt{D}-1}\right]^2 \tag{I.11}$$

where $u_{1-\alpha} = u_\gamma$ may be taken from Table I.2.

Step 2

Compute a tentative c value, c_t, from Equation (I.3).

Step 3

If $\mu_{0t} \leq 5$, check c_t, D', α' and β' using Equation (I.1). If necessary, change μ_{0t} continuously to achieve reasonable agreement with the specified values.

Step 4

Compute n or t_t^* using $n = \dfrac{\mu_{0t}}{\lambda_0 t_t^*}$ *or* $t_t^* = \dfrac{\mu_{0t}}{\lambda_0 n}$ respectively.

I.3.2.7 Determination of *D* and *c* for $\alpha \neq \beta$

Specified: μ_0 , α and β, ($\alpha \neq \beta$)

To be derived: *D*, *c*

Procedures

Step 1

Find the average risk, γ, from $\gamma = \dfrac{\alpha + \beta}{2}$

Step 2

Proceed according to I.3.2.1 or I.3.2.4.

Step 3

Using the OC curves, by the procedures of I.3.2.2 or I.3.2.3 check whether the derived *c* value gives α' and β' values in reasonable agreement with the specified α and β values, otherwise, increase or decrease *c* by 1.

I.3.3 Tables of the cumulative normal distribution and its inverse

Table I.1 – Cumulative normal distribution for fixed u_γ value

u_γ	2,5	2,0	1,5	1,25	1,0	0,5	0,0
$\Phi\ (u_\gamma) = \gamma$	0,993 8	0,977 3	0,933 2	0,894 4	0,841 3	0,691 5	0,500 0
$1-\gamma$	0,006 2	0,022 7	0,066 8	0,105 6	0,158 7	0,308 5	0,500 0
NOTE Notice that $u_\gamma = -u_{1-\gamma}$.							

Table I.2 – Inverse cumulative normal distribution for fixed $1-\gamma$ values

$1-\gamma$	0,01	0,025	0,05	0,10	0,15	0,20	0,30
$\Phi\ (u_\gamma) = \gamma$	0,99	0,975	0,95	0,90	0,85	0,80	0,70
u_γ	2,326	1,960	1,645	1,282	1,036 4	0,841 6	0,524 4

Annex J
(informative)

Examples and mathematical references for the calendar time terminated test plans

NOTE 1 See Clause 9.

NOTE 2 This Annex uses the symbols listed in 3.2.2.

J.1 Examples

J.1.1 Example 1

Specified: A non-repaired item has a mean time to failure = m_0 = 2 000 h, corresponding to a failure rate $\lambda_0 = \frac{1}{m_0} = 500 \times 10^{-6}\,\text{h}^{-1}$ = 4,4 /year.

A number of items, n, is placed on a fixed calendar time test for $t^*_{\text{cal,t}}$ about 500 h (3 weeks). The producer and the customer accept $\alpha = \beta$ = 5 % and D = 2.

To be derived: p_0, n, c and final test time $t^*_{\text{cal,t}}$

Procedures: According to 9.4, $p_0 = 1 - \exp(\frac{-t^*_{\text{cal,t}}}{m_0}) = 1 - \exp(\frac{-500}{2\,000}) = 0{,}221$. This value is not indicated in IEC 61123, so $t^*_{\text{cal,t}}$ needs to be adjusted. This is done using Equation (2) given in 9.4.1 with p_0 = 0,20: $t^*_{\text{cal,t}} = -m_0 \ln(1 - p_0)$ = 446 h.

In Table 2 of IEC 61123, the following test plan is found for q_0 = 0,80 or p_0 = 0,2; $\alpha = \beta$ = 5 %; D = 2:

n = 60 with c = 17

The final fixed calendar test time $t^*_{\text{cal,t}}$ = 446 h is close to the specified 500 h.

J.1.2 Example 2

Specified: The same data as in example 1, except that the maximum available number of items is n = 210, and the test time is to be determined.

To be derived: p_0, final n, c and $t^*_{\text{cal,t}}$

Procedure: In Table 2 of IEC 61123, the following test plan is found for $\alpha = \beta$ = 5 %; D = 2 and n less than and close to the specified 210:

n = 203 with c = 20 and q_0 = 0,93 corresponding to $p_0 = 1 - q_0$ = 0,07.

Using Equation (2) given in 9.4.1, the fixed calendar test time is determined as follows:

$$t^*_{\text{cal,t}} = -m_0 \ln(1 - p_0) = -2\,000 \ln(0{,}93) = 145 \text{ h}.$$

J.2 Mathematical background

Formula (1) given in 9.4.1 corresponds to the reliability $R(t) = 1 - p_0 = \exp(\frac{-t}{m_0})$ for exponentially distributed time to failure, where t is the calendar time. In cases where the number of failing items is small compared to the number of items under test, such as $\frac{r}{n} < 0{,}1$, the procedures in Clause 8 may be used.

Annex K
(informative)

Derivation and mathematical reference for the optimized test plans of GOST 27.402

K.1 List of symbols used in Annex K

NOTE The terminology in Annex K is different from the terminology listed in 3.2.2.

A	operating parameter
a_k	number of the minimal interior point in the k-section
B	operating parameter
b_k	number of the maximal interior point in the k-section
i	summation index in the k-section
j	number of the horizontal section (for the acceptable boundary points), $j = 0,1,\dots, R$-1
k	number of the vertical section, $k = 1,2,\dots, s$
l or l	number of the horizontal section (for the interior points)
m	summation index in the (k+1)-section
P_a	probability of acceptance (OC-curve)
p_j	probability of passing through point on the acceptable boundary
$Q_i^{(k)}$	probability of an output of F-lines among the rejection boundary from all interior points of the plan
q_i	probability of passing through inner point
$q_l^{(k)}$	probabilities of F-lines passage through all acceptable boundary points
R	maximum (reject) number of failures (operating parameter)
s	summation variable
T	the unknown MTTF or MTBF
T_i	abscissa for vertical lines passing cross points of acceptable boundaries with horizontal lines
$T_{\max}$	maximum test time (operating parameter)
T_β	the specified unacceptable (reject) value of T
T_α	the specified acceptable value of T
T_e^*	mean (expected) test time to acceptance or reject decision
$T_e^*(+)$	mean (expected) test time to acceptance decision
t_k	test time to the k-section
$t_1 \dots t_k \dots t_s$	abscissa for vertical lines passing cross points of acceptable and reject boundaries with horizontal lines
α	nominal producer's risk (type 1 risk)
α_1	true producer's risk

β nominal consumer's risk (type 2 risk)

β_1 true consumer's risk

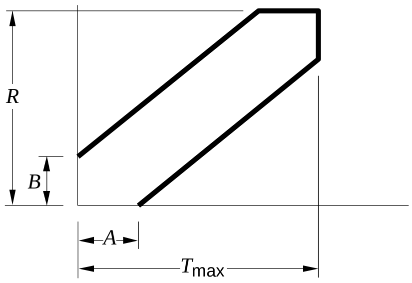
λ the unknown failure rate

$\tilde{\Delta}_{k+1,i}$ mean (expected) test time from i horizontal of k-section to reject boundary during test time interval Δ_{k+1}

Δ_k k-*th* test time interval, $\Delta_k = t_k - t_{k-1}$

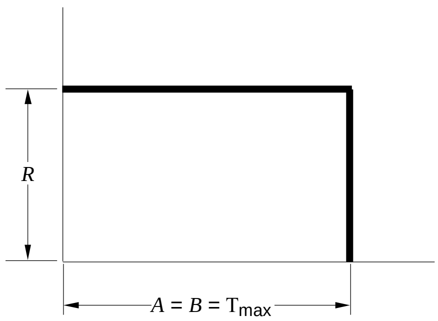
$\tau_1 \dots \tau_j \dots \tau_{R-1}$ abscissa for vertical lines passing cross points of acceptable and horizontal lines (test time to acceptance decision with j failures)

K.2 Test plan types and terminology



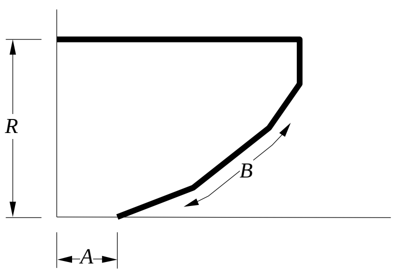
Truncated sequential test plan
The figure shows the operating parameters of a T/S plan (A, B, R, T_{max}).

These test plans are found in Table 2 and Annexes A and D.



Time/failure terminated test plan (T/F)
The figure shows the operating parameters of a T/F plan (A and R).

These test plans can be found in Table 3 and Annex B.



Combined test plan from GOST 27.402
The figure shows the operating parameters of a combined test plan (A, B, R).
The operating parameter B causes a gradual change of the acceptance boundary and does not have a graphic analogue.

These test plans can be found in Annex D.

Figure K.1 – Test plan types and terminology

K.3 General introduction

In these plans, the operating characteristic is:

$$P_\mathrm{a} = \sum_{j=0}^{R-1} p_j \tag{K.1}$$

and the risks are:

$$\alpha = 1 - P_\mathrm{a}(T_\alpha) = 1 - \sum_{j=0}^{R-1} p_j(T_\alpha) \tag{K.2}$$

and

$$\beta = P_\mathrm{a}(T_\beta) = \sum_{j=0}^{R-1} p_j(T_\beta) \tag{K.3}$$

Values of the mean test time to acceptance decision $T_\mathrm{e}^*(+)$ in the points T (for example $T = T_\alpha$ *and* $T = T_\beta$) are computed using the following:

$$T_\mathrm{e}^*(+) = \frac{\sum_{j=0}^{R-1} \tau_j p_j}{\sum_{j=1}^{R-1} p_j} \tag{K.4}$$

Values of mean test time T_e^* in the points T (for example $T = T_\alpha$ and $T = T_\beta$) are computed using the following equation:

$$T_e^* = \sum_{j=0}^{R-1} p_j \tau_j + \sum_{k=1}^{s-1} \sum_{i=a_k}^{b_k} q_i^{(k)} Q_i^{(k)} \left\{ t_k + \underbrace{\frac{b_{k+1} - i + 1}{\lambda Q_i^{(k)}} \left[1 - \mathrm{e}^{-\lambda \Delta_{k+1}} \sum_{m=0}^{b_{k+1}-i+1} (\lambda \Delta_{k+1})^m \Big/ m! \right]}_{\tilde{\Delta}_{k+1,i}} \right\} \tag{K.5}$$

where $Q_i^{(k)} = 1 - \mathrm{e}^{-\lambda \Delta_{k+1}} \sum_{m=0}^{b_{k+1}-i} \frac{(\lambda \Delta_{k+1})^m}{m!}$

K.4 Procedure used for developing the optimized test plans

The test plans in Annex D are copied from GOST 27.402 where more of these test plan types can be found.

The test plans were developed using the following iterative procedure.

Step 1 (Preamble and preparation)

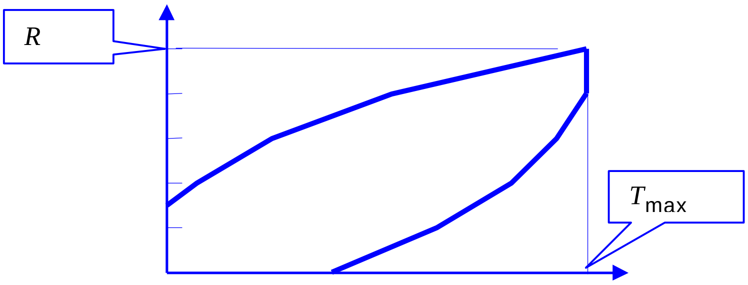


Figure K.2 – Principle of test plans

The characteristics of a common form of the plan with the free arbitrary acceptable and reject boundaries, maximum reject failures R and maximum test time T_{max} shall be calculated.

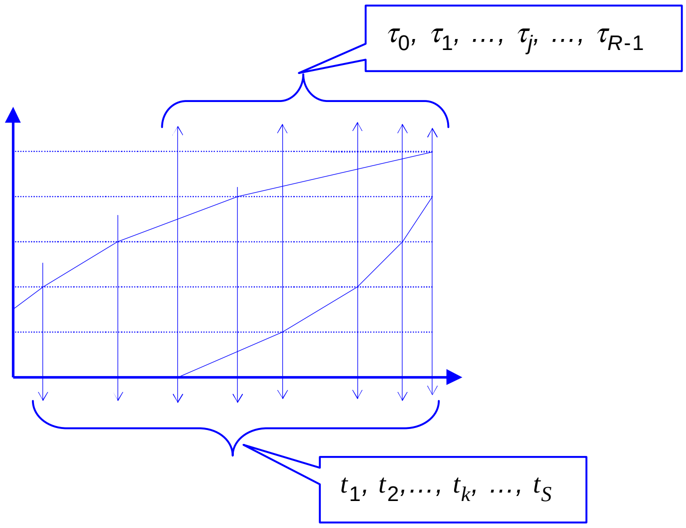


Figure K.3 – Partitioning of the test plan graph

a) Vertical lines passing through cross points of acceptable and reject boundaries with horizontal lines are t. Vertical lines passing through cross points of acceptable boundaries with horizontal lines are τ.

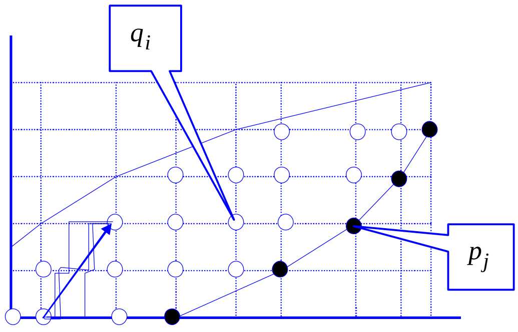


Figure K.4 – Interior nodes and border nodes

b) Vertical and horizontal lines intersect at white points in the continue testing region. The points where accept boundary and horizontal lines intersect are black. To calculate test plan characteristics it is sufficient to examine only the black points.

All the possible failure realization lines (F-lines) between two points of adjacent sections are shown by one arrow as illustrated in Figure K.5:

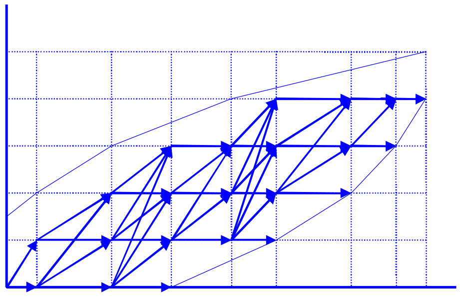


Figure K.5 – Paths to the accept line

c) Arrows show all the possible transitions of F-lines at interior points to the acceptable boundary:

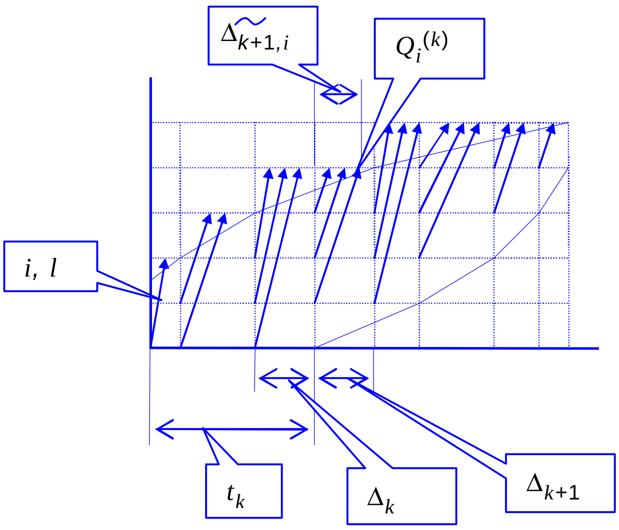


Figure K.6 – Paths to the reject line

d) Arrows show all the possible transitions of F-lines at interior points to the reject boundary.

Step 2

Choose the necessary values of the initial data: *D*, α and *β* .

Step 3

Choose the type of plan, for example A-type plan (see Figure K.1).

Step 4

Determine the starting values of operating parameters A, B, R, T_{max} for the first plan, The starting values A and B are calculated with the help of the Abraham Wald equations, The starting values R and T_{max} are calculated the same way as for T/F plan, Thus boundaries of the plan coincide with boundaries of A-type plans. This is used as a starting point for developing the test plans of Annex D.

Step 5

Calculate:

a) probabilities of F-lines passage through all internal (q) points of the plan using Equation (K.6):

$$q_l^{(k)} = \mathrm{e}^{-\lambda\Delta_k} \sum_{i=a_{k-1}}^{\min(l,b_{k-1})} q_i^{(k-1)} \frac{(\lambda\Delta_k)^{l-i}}{(l-i)!} \tag{K.6}$$

and probabilities of F-lines passage through all acceptable boundary *(p)* points of the plan using Equation (K.7):

$$p_j \equiv q_{l=a_{k-1}}^{(k)} = q_{a_{k-1}}^{(k-1)} e^{-\lambda\Delta_k} \tag{K.7}$$

Equation (K.7) is a special case of Equation (K.6) at i = l

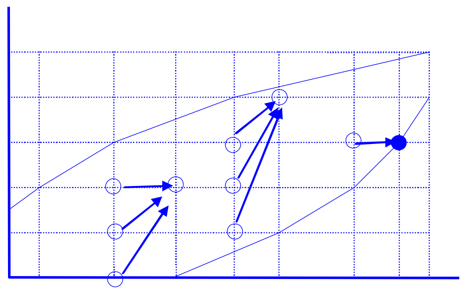


Figure K.7 – Probabilities of paths transfer between nodes

The probabilities q, p and $Q_i^{(k)}$; L, $\tilde{\Delta}_{k+1,i}$, T_e^*, $T_\mathrm{e}^*(+)$ are the functions of the unknown value T.

Equations (K.6) and (K.7) are recurrent (identical to all points of the plan). Calculations under this formula are carried out for each point consistently on vertical sections and from below upward (it is possible from the top downward, but it is less convenient).

b) values of the operating characteristic P_a including true risks alpha and beta are computed using Equation (K.8):

$$P_\mathrm{a} = \sum_{j=0}^{R-1} p_j \tag{K.8}$$

Particularly

$$\alpha = 1 - P_{\mathrm{a}}(T_\alpha) = 1 - \sum_{j=0}^{R-1} p_j(T_\alpha) \qquad \text{(K.9)}$$

$$\beta = P_{\mathrm{a}}(T_\beta) = \sum_{j=0}^{R-1} p_j(T_\beta) \qquad \text{(K.10)}$$

c) values of the mean test time to acceptance decision $T_{\mathrm{e}}^{*}(+)$ in the points T (for example $T = T_\alpha$ *and* $T = T_\beta$) are computed using Equation (K.11):

$$T_{\mathrm{e}}^{*}(+) = \frac{\sum_{j=0}^{R-1} \tau_j p_j}{\sum_{j=1}^{R-1} p_j} \qquad \text{(K.11)}$$

d) values of mean test time T_{e}^{*} in the points T (for example $T = T_\alpha$ and $T = T_\beta$) are computed using Equation (K.12):

$$T_{\mathrm{e}}^{*} = \sum_{j=0}^{R-1} p_j \tau_j + \sum_{k=1}^{s-1} \sum_{i=\alpha_k}^{b_k} q_i^{(k)} Q_i^{(k)} \left\{ t_k + \frac{b_{k+1} - i + 1}{\lambda Q_i^{(k)}} \left[1 - \mathrm{e}^{-\lambda \Delta_{k+1}} \sum_{m=0}^{b_{k+1}-i+1} (\lambda \Delta_{k+1})^m \Big/ m! \right] \right\} \qquad \text{(K.12)}$$

where

$$Q_i^{(k)} = 1 - \mathrm{e}^{-\lambda \Delta_{k+1}} \sum_{m=0}^{b_{k+1}-i} \frac{(\lambda \Delta_{k+1})^m}{m!}$$

The starting parameter values under $k = 0$*:* $t_0 = 0$*,* $q_0^{(0)} = 1$, $q_i^{(0)} = 0$

The probabilities $Q_i^{(k)}$ of an output of F-lines along the rejection boundary from all internal points of the plan and corresponding mean test times $\widetilde{\Delta}_{k+1,i}$ in interval Δ_{k+1} are shown in Figure K.6.

In Annex D values of these characteristics are calculated in 15 points including $T = T_\alpha$.

The result of calculation is a plan for which values α_1 and β_1 do not coincide with nominal values α and β, and the value T_{e}^{*} and/or $T_{\mathrm{e}}^{*}(+)$ is not the minimal possible one.

Step 6

Select new values of operating parameters A and B for inclined boundaries of the plan and compute another plan with other α_1 and β_1 values.

Step 7

Repeat the procedure of step 6 until true values of risks will coincide with nominal values $\alpha_1 = \alpha$ and $\beta_1 = \beta$ with the necessary accuracy.

NOTE Risks of plans are given in Annex D to 4 decimal places, and calculated with the help of a computer to 6 decimal places.

The result is an accurate plan. However, it is still not the optimum plan.

Step 8

Change the value of the operating parameter T_{max} and, if it is necessary, R, and repeat the procedure of steps 6 to 7 and compute another accurate plan with another value of T_e^*.

NOTE The number of accurate test plans with various values T_e^* is infinite.

Step 9

Repeat steps 6 to 8 until the plan with the minimal T_e^* or $T_e^*(+)$ are found within the limits of allowable values and with the necessary accuracy.

NOTE 1 Values of the T_e^* and $T_e^*(+)$ plans characteristics are given in Annex D to 4 decimal places, and calculated with the help of a computer to 6 decimal places.

Such a plan is accurate and optimum.

NOTE 2 Recursive equations in a full form (for example Equation (K.12)), are frequently unwieldy. They have an illustrative character and are not user-friendly. If the user wants to carry out manual calculations, he should "take the formula to pieces", identify a recursive element and use it for calculating in all points of the plan. The computer program also uses a recursion.

K.5 The recurrent element

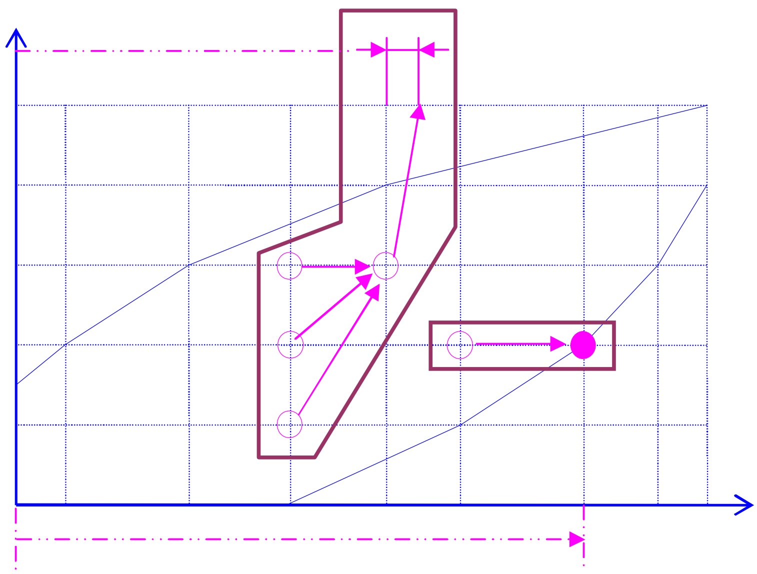


Figure K.8 – The recurrent element – Two cases

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